1. Risolvi:

$$\sin 23° + \sin(-23°) = \mathbf{0} \qquad \tan 77° = k \quad \Rightarrow \quad \cot 13° = \boldsymbol{k}$$

$$\sin 89° = \boldsymbol{\cos}\ 1° \qquad \cos 263° = k \quad \Rightarrow \quad \sin 7° = \boldsymbol{-k}$$

$$\sin 5° - \sin 175° + \cos 5° + \cos 175° = \mathbf{0} \qquad \tan\left(\frac{3}{2}\pi - \alpha\right)\sin(\pi - \alpha) = \boldsymbol{\cos\alpha}$$

Infatti, usando le formule degli archi associati, otteniamo:

$$\sin 23° + \sin(-23°) = \sin 23° - \sin 23° = 0 \qquad \cot 13° = \cot(90° - 77°) = \tan 77°$$

$$\sin 89° = \sin(90° - 1°) = \cos 1° \qquad \cos 263° = \cos(270° - 7°) = -\sin 7°$$

$$\sin 5° - \sin(180° - 5°) + \cos 5° + \cos(180° - 5°) = \sin 5° - \sin 5° + \cos 5° - \cos 5° = 0$$

$$\tan\left(\frac{3}{2}\pi - \alpha\right)\sin(\pi - \alpha) = \cot\alpha \sin\alpha = \frac{\cos\alpha}{\sin\alpha}\sin\alpha = \cos\alpha$$

2. Disegna i grafici delle seguenti funzioni e indica il loro periodo:

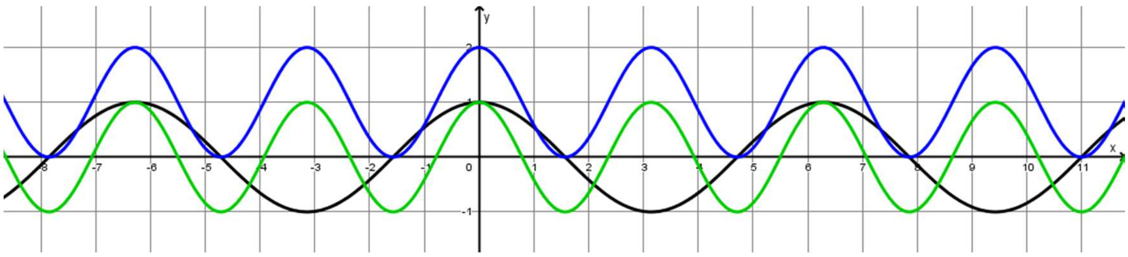
$$y = \cos^2 x - \sin^2 x + 1 \qquad y = \cos x - \sin x$$

$$y = \cos^2 x - \sin^2 x + 1 = \cos 2x + 1 \qquad T = \frac{2\pi}{2} = \boldsymbol{\pi}$$

$y = \cos x$

$y = \cos 2x$

$y = \cos 2x + 1$

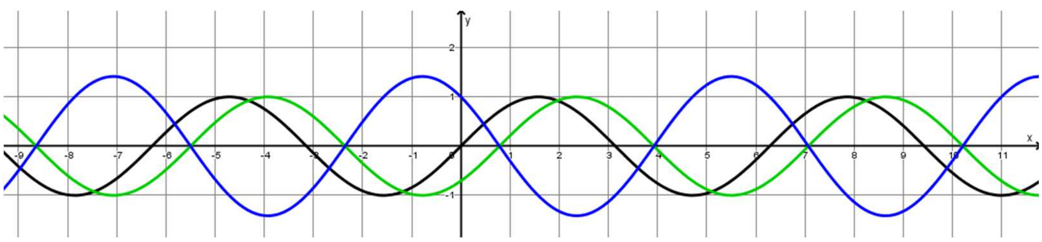


$$y = \cos x - \sin x = \sqrt{2}\left(\frac{\sqrt{2}}{2}\sin x - \frac{\sqrt{2}}{2}\cos x\right) = -\sqrt{2}\sin\left(x - \frac{\pi}{4}\right) \qquad T = \frac{2\pi}{1} = \boldsymbol{2\pi}$$

$y = \sin x$

$y = \sin\left(x - \frac{\pi}{4}\right)$

$y = -\sqrt{2}\sin\left(x - \frac{\pi}{4}\right)$



Semplifica le seguenti espressioni:

3. $-\cos\,(240° - \alpha)\sin\,(60° + \alpha) + \cos\,(60° + \alpha)\sin(240° - \alpha) - 2\sin\alpha\cos\alpha$

$$= \sin\big(240° - \alpha - (60° + \alpha)\big) - \sin 2\alpha = \sin(180° - 2\alpha) - \sin 2\alpha = \sin 2\alpha - \sin 2\alpha = \mathbf{0}$$

4. $\cot^2\alpha - \sec^2\left(\frac{\pi}{2} - \alpha\right)(\cos^2\alpha + 1)^2 + 3\csc^2\alpha - 2 + \cos^2\left(\frac{3}{2}\pi + \alpha\right)$

$$= \cot^2\alpha - \csc^2\alpha\;(\cos^4\alpha + 2\cos^2\alpha + 1) + 3\csc^2\alpha - 2 + \sin^2\alpha =$$

$$= \cot^2\alpha - \frac{\cos^4\alpha}{\sin^2\alpha} - \frac{2\cos^2\alpha}{\sin^2\alpha} - \csc^2\alpha + 3\csc^2\alpha - 2 + \sin^2\alpha =$$

$$= \cot^2\alpha - \cot^2\alpha \cdot \cos^2\alpha - 2\cot^2\alpha + 2\csc^2\alpha - 2 + \sin^2\alpha =$$

$$= \cot^2\alpha\;(1 - \cos^2\alpha) - \frac{2\cos^2\alpha}{\sin^2\alpha} + \frac{2}{\sin^2\alpha} - 2 + \sin^2\alpha =$$

$$= \frac{\cos^2\alpha}{\sin^2\alpha} \cdot \sin^2\alpha + 2\frac{-\cos^2\alpha + 1}{\sin^2\alpha} - 2 + \sin^2\alpha =$$

$$= \cos^2\alpha + \frac{2\sin^2\alpha}{\sin^2\alpha} - 2 + \sin^2\alpha = (\cos^2\alpha + \sin^2\alpha) + 2 - 2 = \mathbf{1}$$

5. $\tan\frac{\alpha}{2}\;[\cos\pi + \cos\,(\pi - \alpha)] \cdot \sin\alpha + 4\cos\left(\alpha - \frac{\pi}{3}\right)\sin\left(\frac{\pi}{6} - \alpha\right) + 5\sin^2\alpha + 1$

$$= \frac{\sin\alpha}{1 + \cos\alpha}(-1 - \cos\alpha) \cdot \sin\alpha + 4\left(\frac{1}{2}\cos\alpha + \frac{\sqrt{3}}{2}\sin\alpha\right)\left(\frac{1}{2}\cos\alpha - \frac{\sqrt{3}}{2}\sin\alpha\right) + 5\sin^2\alpha + 1 =$$

$$= -\frac{\sin\alpha}{1 + \cos\alpha}(1 + \cos\alpha) \cdot \sin\alpha + 4\left(\frac{1}{4}\cos^2\alpha - \frac{3}{4}\sin^2\alpha\right) + 5\sin^2\alpha + 1 =$$

$$= -\sin^2\alpha + \cos^2\alpha - 3\sin^2\alpha + 5\sin^2\alpha + 1 = \sin^2\alpha + \cos^2\alpha + 1 = 1 + 1 = \mathbf{2}$$

6. $\frac{\cos 2\alpha}{\sin^2\left(\frac{\pi}{2} + \alpha\right)} - \frac{2\tan\alpha}{\tan 2\alpha} + 3$

$$= \frac{\cos^2\alpha - \sin^2\alpha}{\cos^2\alpha} - \frac{2\tan\alpha}{\frac{2\tan\alpha}{1 - \tan^2\alpha}} + 3 =$$

$$= \frac{\cos^2\alpha}{\cos^2\alpha} - \frac{\sin^2\alpha}{\cos^2\alpha} - 2\tan\alpha \cdot \frac{1 - \tan^2\alpha}{2\tan\alpha} + 3 =$$

$$= 1 - \tan^2\alpha - 1 + \tan^2\alpha + 3 = \mathbf{3}$$

Verifica le seguenti identità:

7. $\sin\alpha\tan\frac{\alpha}{2}+\cos^2\frac{\alpha}{2}=2-\cos\alpha-\sin^2\frac{\alpha}{2}$

$$\sin\alpha\cdot\frac{1-\cos\alpha}{\sin\alpha}+\cos^2\frac{\alpha}{2}+\sin^2\frac{\alpha}{2}=2-\cos\alpha$$

$$1-\cos\alpha+1=2-\cos\alpha \qquad 2-\cos\alpha=2-\cos\alpha$$

8. $\frac{1-\sin 2\alpha}{\cos 2\alpha}=\frac{1-\tan\alpha}{1+\tan\alpha}$

$$\frac{1-2\sin\alpha\cos\alpha}{\cos^2\alpha-\sin^2\alpha}=\frac{1-\frac{\sin\alpha}{\cos\alpha}}{1+\frac{\sin\alpha}{\cos\alpha}} \qquad \frac{1-2\sin\alpha\cos\alpha}{(\cos\alpha-\sin\alpha)(\cos\alpha+\sin\alpha)}=\frac{\cos\alpha-\sin\alpha}{\cos\alpha+\sin\alpha}$$

$$\frac{1-2\sin\alpha\cos\alpha}{\cos\alpha-\sin\alpha}=\cos\alpha-\sin\alpha \qquad 1-2\sin\alpha\cos\alpha=(\cos\alpha-\sin\alpha)^2$$

$$1-2\sin\alpha\cos\alpha=\cos^2\alpha+\sin^2\alpha-2\sin\alpha\cos\alpha \qquad 1-2\sin\alpha\cos\alpha=1-2\sin\alpha\cos\alpha$$

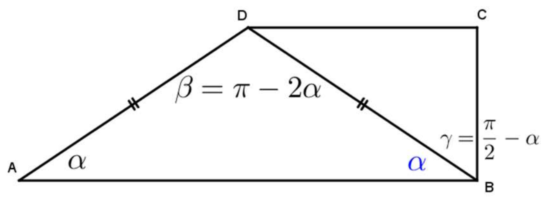
9. Determina $\tan\left(\alpha-\frac{\pi}{4}\right)$ sapendo che $\cos\alpha=\frac{1}{3}$ con $0<\alpha<\frac{\pi}{2}$.

$$\cos\alpha=\frac{1}{3} \qquad \sin\alpha=\sqrt{1-\cos^2\alpha}=\sqrt{1-\frac{1}{9}}=\frac{2\sqrt{2}}{3} \qquad \tan\alpha=\frac{\sin\alpha}{\cos\alpha}=\frac{\frac{2\sqrt{2}}{3}}{\frac{1}{3}}=2\sqrt{2}$$

$$\tan\left(\alpha-\frac{\pi}{4}\right)=\frac{\tan\alpha-\tan\frac{\pi}{4}}{1+\tan\alpha\tan\frac{\pi}{4}}=\frac{2\sqrt{2}-1}{1+2\sqrt{2}}\cdot\frac{2\sqrt{2}-1}{2\sqrt{2}-1}=\frac{8+1-4\sqrt{2}}{8-1}=\boldsymbol{\frac{9-4\sqrt{2}}{7}}$$

10. Nel trapezio rettangolo disegnato qui sotto, sapendo che $\sin\alpha=\frac{4}{5}$, determina $\sin\beta$ e $\cos\beta$, $\sin\gamma$ e $\cos\gamma$.

$$\sin\alpha=\frac{4}{5} \qquad \cos\alpha=\sqrt{1-\sin^2\alpha}=\sqrt{1-\frac{16}{25}}=\frac{3}{5}$$



Siccome il triangolo ABD è isoscele, gli angoli alla base sono congruenti, come indicato nella figura. Perciò l'angolo al vertice β, sapendo che la somma degli angoli interni di un triangolo è 180°, è dato dalla differenza tra 180° e i due angoli alla base, cioè:

$$\beta=\pi-2\alpha \quad\Rightarrow\quad \sin\beta=\sin(\pi-2\alpha)=\sin 2\alpha=2\sin\alpha\cos\alpha=2\cdot\frac{4}{5}\cdot\frac{3}{5}=\boldsymbol{\frac{24}{25}}$$

$$\cos\beta=\cos^2\alpha-\sin^2\alpha=\frac{9}{25}-\frac{16}{25}=\boldsymbol{-\frac{7}{25}}$$

Il triangolo BCD è rettangolo in C e, trattandosi di un trapezio rettangolo, γ è complementare di α:

$$\sin\gamma=\sin\left(\frac{\pi}{2}-\alpha\right)=\cos\alpha=\boldsymbol{\frac{3}{5}} \qquad \cos\gamma=\cos\left(\frac{\pi}{2}-\alpha\right)=\sin\alpha=\boldsymbol{\frac{4}{5}}$$