1. $\frac{\sqrt{8+2\sqrt{7}}\,(\sqrt{7}-1)}{\sqrt{6}}-\left(\sqrt{2}+\sqrt{3}\right)^2+\sqrt{3}\left(\sqrt{2}+6\right)+5$

$$=\frac{\sqrt{\left(1+\sqrt{7}\right)^2}\left(\sqrt{7}-1\right)}{\sqrt{6}}-\left(2+3+2\sqrt{6}\right)+\sqrt{6}+6\sqrt{3}+5=$$

$$=\frac{\left(\sqrt{7}+1\right)\left(\sqrt{7}-1\right)}{\sqrt{6}}-5-2\sqrt{6}+\sqrt{6}+6\sqrt{3}+5=\frac{7-1}{\sqrt{6}}\cdot\frac{\sqrt{6}}{\sqrt{6}}-\sqrt{6}+6\sqrt{3}=\sqrt{6}-\sqrt{6}+6\sqrt{3}=\mathbf{6\sqrt{3}}$$

2. $\left(\frac{5-2\sqrt{6}}{3}\cdot\frac{1}{\sqrt{3}+\sqrt{2}}+\frac{\sqrt{128}}{3}\right)^2+\left(\sqrt{6}+3\right)^2$

$$=\left(\frac{5-2\sqrt{6}}{3\left(\sqrt{3}+\sqrt{2}\right)}\cdot\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}}+\frac{8\sqrt{2}}{3}\right)^2+6+9+6\sqrt{6}=$$

$$=\left(\frac{5\sqrt{3}-5\sqrt{2}-6\sqrt{2}+4\sqrt{3}+8\sqrt{2}}{3}\right)^2+15+6\sqrt{6}=$$

$$=\left(3\sqrt{3}-\sqrt{2}\right)^2+15+6\sqrt{6}=27+2-6\sqrt{6}+15+6\sqrt{6}=\mathbf{44}$$

3. $\left(\sqrt{18}+\sqrt{32}-\sqrt{50}\right)\left(\sqrt{2}-1\right)+\left(\sqrt{\sqrt[3]{32}}\right)^3-\sqrt{2}\left(2\sqrt{2}-1\right)$

$$=\left(3\sqrt{2}+4\sqrt{2}-5\sqrt{2}\right)\left(\sqrt{2}-1\right)+\sqrt{32}-4+\sqrt{2}=$$

$$=2\sqrt{2}\left(\sqrt{2}-1\right)+4\sqrt{2}-4+\sqrt{2}=4-2\sqrt{2}+4\sqrt{2}-4+\sqrt{2}=\mathbf{3\sqrt{2}}$$

4. $\left(6+2\sqrt{5}\right)\sqrt{6-2\sqrt{5}}-3\left(\sqrt{5}+2\sqrt{2}\right)+\left(3\sqrt{2}+1\right)\left(2-\sqrt{5}\right)+3\sqrt{10}$

$$=\left(6+2\sqrt{5}\right)\sqrt{\left(\sqrt{5}-1\right)^2}-3\sqrt{5}-6\sqrt{2}+6\sqrt{2}-3\sqrt{10}+2-\sqrt{5}+3\sqrt{10}=$$

$$=\left(6+2\sqrt{5}\right)\left(\sqrt{5}-1\right)-4\sqrt{5}+2=$$

$$=6\sqrt{5}-6+10-2\sqrt{5}-4\sqrt{5}+2=\mathbf{6}$$

5. $\frac{5}{3}\left[20-\left(\sqrt{2}-3\sqrt{3}\right)^2\right]:\left(2\sqrt{6}+3\right)+\frac{25}{11-4\sqrt{6}}$

$$=\frac{5}{3}\left[20-\left(2+27-6\sqrt{6}\right)\right]:\left(2\sqrt{6}+3\right)+\frac{25}{11-4\sqrt{6}}\cdot\frac{11+4\sqrt{6}}{11+4\sqrt{6}}=$$

$$=\frac{5}{3}\frac{\left[-9+6\sqrt{6}\right]}{2\sqrt{6}+3}+\frac{25\left(11+4\sqrt{6}\right)}{121-96}=$$

$$=5\frac{-3+2\sqrt{6}}{2\sqrt{6}+3}\cdot\frac{2\sqrt{6}-3}{2\sqrt{6}-3}+11+4\sqrt{6}=5\frac{24+9-12\sqrt{6}}{24-9}+11+4\sqrt{6}=11-4\sqrt{6}+11+4\sqrt{6}=\mathbf{22}$$

6. $2\sqrt{\sqrt{x+4}+2}\cdot\sqrt{\sqrt{x+4}-2}\,(\sqrt{x+8}+\sqrt{x})-\left(\sqrt{x^2+8x}+1\right)^2+(x+4+\sqrt{7})(x+4-\sqrt{7})$

$= 2\sqrt{x+4-4}\,(\sqrt{x+8}+\sqrt{x})-\left(x^2+8x+1+2\sqrt{x^2+8x}\right)+x^2+8x+16-7=$ $\quad C.E.:\begin{cases}x+4\geq 0\\ x+8\geq 0\\ x\geq 0\end{cases}\quad x\geq 0$

$= 2\sqrt{x^2+8x}+2x-x^2-8x-1-2\sqrt{x^2+8x}+x^2+8x+9=\mathbf{2x+8}$

7. $\frac{\sqrt{x}}{\sqrt{x}-\sqrt{x-2}}-\frac{\sqrt{x-2}}{\sqrt{x}+\sqrt{x-2}}$

$=\frac{x+\sqrt{x^2-2x}-\sqrt{x^2-2x}+x-2}{x-(x-2)}=\frac{2x-2}{2}=\mathbf{x-1}$ $\quad C.E.:\begin{cases}x\geq 0\\ x-2\geq 0\end{cases}\quad x\geq 2$

8. $\left[\left(a-\sqrt{ab}\right)^2-(\sqrt{b}-\sqrt{a})(a\sqrt{b}+b\sqrt{a})-a\,(a+b)\right](a-b)-\left(b\,\sqrt[4]{ab}\right)^2$

$=\left[a^2+ab-2a\sqrt{ab}-\left(ab+b\sqrt{ab}-a\sqrt{ab}-ab\right)-a^2-ab\right](a-b)-b^2\sqrt{ab}=$ $\quad C.E.: a\geq 0\ \wedge\ b\geq 0$

$=\left(-2a\sqrt{ab}-b\sqrt{ab}+a\sqrt{ab}\right)(a-b)-b^2\sqrt{ab}=$

$=-\sqrt{ab}\,(a+b)(a-b)-b^2\sqrt{ab}=-a^2\sqrt{ab}+b^2\sqrt{ab}-b^2\sqrt{ab}=\mathbf{-a^2\sqrt{ab}}$

9. $\left(\sqrt{a\,\sqrt[3]{a^3}}:\ \sqrt[4]{a^4\,\sqrt{a}}+\frac{1}{\sqrt[8]{a}}\right)^2-\frac{2}{\sqrt[4]{a}}$

$=\left(\sqrt{a^2}:\left(a\,\sqrt[8]{a}\right)+\frac{1}{\sqrt[8]{a}}\right)^2-\frac{2}{\sqrt[4]{a}}=$ $\quad C.E.: a>0$

$=\left(\frac{a}{a\sqrt[8]{a}}+\frac{1}{\sqrt[8]{a}}\right)^2-\frac{2}{\sqrt[4]{a}}=\left(\frac{2}{\sqrt[8]{a}}\right)^2-\frac{2}{\sqrt[4]{a}}=\frac{4}{\sqrt[4]{a}}-\frac{2}{\sqrt[4]{a}}=\frac{2}{\sqrt[4]{a}}\cdot\frac{\sqrt[4]{a^3}}{\sqrt[4]{a^3}}=\mathbf{\frac{2}{a}\sqrt[4]{a^3}}$

10. $\left[\left(a\sqrt{a}-1\right)^2+\left(a+\sqrt{a}\right)^2-1-a^3-2a\right]^2:\left[a\left(1-\sqrt{a}\right)\right]^2$

$=\frac{\left(a^3+1-2a\sqrt{a}+a^2+a+2a\sqrt{a}-1-a^3-2a\right)^2}{a^2\left(1+a-2\sqrt{a}\right)}=$ $\quad C.E.:\begin{cases}a>0\\ a\neq 1\end{cases}$

$=\frac{(a^2-a)^2}{a^2\,\left(1+a-2\sqrt{a}\right)}\cdot\frac{1+a+2\sqrt{a}}{1+a+2\sqrt{a}}=$

$=\frac{a^2(a-1)^2}{a^2\,(a^2+1+2a-4a)}\left(1+a+2\sqrt{a}\right)=$

$=\frac{(a-1)^2}{(a-1)^2}\left(1+a+2\sqrt{a}\right)=\mathbf{1+a+2\sqrt{a}}$

11. In un rettangolo la base misura $(3\sqrt{5}+1)\ cm$ e l'area $(17+7\sqrt{5})\ cm^2$. Aumentando di $(\sqrt{5}+4)\ cm$ la base e di $(\sqrt{5}+6)\ cm$ l'altezza, quanto vale il rapporto tra base e altezza?

Determino innanzi tutto la misura dell'altezza, dividendo l'area per la base, visto che l'area di un rettangolo si calcola facendo il prodotto di base per altezza:

$$h=\frac{A}{b}=\frac{17+7\sqrt{5}}{3\sqrt{5}+1}\cdot\frac{3\sqrt{5}-1}{3\sqrt{5}-1}=\frac{51\sqrt{5}-17+105-7\sqrt{5}}{45-1}=\frac{44\sqrt{5}+88}{44}=\sqrt{5}+2$$

L'altezza misura quindi $(\sqrt{5}+2)\ cm$.

Calcolo il rapporto tra base e altezza, aumentate delle quantità indicate:

$$\frac{3\sqrt{5}+1+\sqrt{5}+4}{\sqrt{5}+2+\sqrt{5}+6}=\frac{4\sqrt{5}+5}{2\sqrt{5}+8}=\frac{\sqrt{5}\left(4+\sqrt{5}\right)}{2\left(\sqrt{5}+4\right)}=\boldsymbol{\frac{\sqrt{5}}{2}}$$