1. $81^x + 4 \cdot (9^x)^2 - 2 \cdot 3^{4x-1} = 117$

$3^{4x} + 4 \cdot (3^{2x})^2 - 2 \cdot \frac{3^{4x}}{3} = 117$ $\qquad$ $3^{4x} + 4 \cdot 3^{4x} - \frac{2}{3} \cdot 3^{4x} = 117$

$3^{4x}\left(1 + 4 - \frac{2}{3}\right) = 117$ $\qquad$ $3^{4x} \cdot \frac{13}{3} = 117$ $\qquad$ $3^{4x} = 117 \cdot \frac{3}{13}$

$3^{4x} = 27$ $\qquad$ $3^{4x} = 3^3$ $\qquad$ $4x = 3$ $\qquad$ $x = \frac{3}{4}$

2. $4^{2x} + 2^{2(x+1)} = 32$

$2^{4x} + 4 \cdot 2^{2x} - 32 = 0$ $\qquad$ Pongo: $2^{2x} = t$

$t^2 + 4t - 32 = 0$ $\qquad$ $t_{1,2} = \frac{-2 \pm \sqrt{4+32}}{1} \left\langle \begin{matrix} -8 \\ 4 \end{matrix} \right.$

$t = 4 \quad \Rightarrow \quad 2^{2x} = 4 \quad \Rightarrow \quad 2^{2x} = 2^2 \quad \Rightarrow \quad 2x = 2 \quad \Rightarrow \quad x = 1$

$t = -8 \quad \Rightarrow \quad 2^{2x} = -8 \quad \Rightarrow \quad$ *imp.*

3. $3^{2x+1} < 5^{-2x-1}$

$3^{2x+1} \cdot 5^{2x+1} < 1$ $\qquad$ $15^{2x+1} < 1$

$15^{2x+1} < 15^0$ $\qquad$ $2x + 1 < 0$ $\qquad$ $x < -\frac{1}{2}$

4. $5^x - 3 \cdot 25^x < 5^{x+1}$

$5^x - 3 \cdot 5^{2x} - 5 \cdot 5^x < 0$ $\qquad$ $-3 \cdot 5^{2x} - 4 \cdot 5^x < 0$

$3 \cdot 5^{2x} + 4 \cdot 5^x > 0$ $\qquad$ $\forall x \in R$

5. $\log_{\sqrt{2}}(2 - x) - \log_{\sqrt{2}}(x - 4) = \log_{\sqrt{2}}(2x - 3)$

$c.a.: \begin{cases} x - 4 > 0 \\ 2x - 3 > 0 \\ 2 - x > 0 \end{cases}$ $\qquad$ $\begin{cases} x > 4 \\ x > \frac{3}{2} \\ x < 2 \end{cases}$ $\qquad$ *imp.*

6. $\log_2 (3x-1) - \log_2 (x+2) = 3$

$c.a.: \begin{cases} 3x-1>0 \\ x+2>0 \end{cases}$ $\quad \begin{cases} x > \frac{1}{3} \\ x > -2 \end{cases}$ $\quad c.a.: x > \frac{1}{3}$

$\log_2 (3x-1) = \log_2 (x+2) + \log_2 8$ $\quad \log_2 (3x-1) = \log_2 [8(x+2)]$

$3x-1 = 8(x+2)$ $\quad 3x-1 = 8x+16$

$5x = -17$ $\quad x = -\frac{17}{5}$ non accettabile per c.a. $\Rightarrow$ *imp.*

7. $2 \ln (x+1) = \ln (5-x)$

$c.a.: \begin{cases} 5-x>0 \\ x+1>0 \end{cases}$ $\quad \begin{cases} x<5 \\ x>-1 \end{cases}$ $\quad c.a.: -1 < x < 5$

$\ln (5-x) = \ln (x+1)^2$ $\quad 5-x = (x+1)^2$

$x^2 + 2x + 1 - 5 + x = 0$ $\quad x^2 + 3x - 4 = 0$

$x_{1,2} = \frac{-3 \pm \sqrt{9+16}}{2} \left\langle \begin{matrix} 1 \\ -4 \end{matrix} \right.$ *non acc. per c.a.* $\quad x = 1$

8. $\log x^3 + 3 = 6 \log x$

$c.a.: x > 0$ $\quad 3 \log x + 3 = 6 \log x$

$\log x + 1 = 2 \log x$ $\quad \log x = 1$ $\quad x = 10$

9. $\log_{\frac{3}{2}} (x^2-1) - \log_{\frac{3}{2}} (x-3) > 1$

$c.a.: \begin{cases} x^2-1>0 \\ x-3>0 \end{cases}$ $\quad \begin{cases} x<-1 \ \vee \ x>1 \\ x>3 \end{cases}$ $\quad c.a.: x > 3$

$\log_{\frac{3}{2}} (x^2-1) > \log_{\frac{3}{2}} (x-3) + \log_{\frac{3}{2}} \frac{3}{2}$ $\quad \log_{\frac{3}{2}} (x^2-1) > \log_{\frac{3}{2}} \left[\frac{3}{2}(x-3) \right]$

$x^2 - 1 > \frac{3}{2}(x-3)$ $\quad 2x^2 - 2 > 3(x-3)$

$2x^2 - 3x + 7 > 0$ $\quad x_{1,2} = \frac{3 \pm \sqrt{9-56}}{4}$ $\quad \Delta < 0 \quad \Rightarrow \quad \forall x \in R$

$\begin{cases} \forall x \in R \\ x > 3 \end{cases} \quad \Rightarrow \quad x > 3$