1. $3^{4x+1} - 82 \cdot 9^x + 27 = 0$

$3 \cdot 3^{4x} - 82 \cdot 3^{2x} + 27 = 0$ Pongo: $3^{2x} = t$

$3t^2 - 82t + 27 = 0$ $t_{1,2} = \dfrac{41 \pm \sqrt{1681 - 81}}{3} \left\langle \begin{array}{l} 27 \\ \dfrac{1}{3} \end{array} \right.$

$t = 27 \quad \Rightarrow \quad 3^{2x} = 27 \quad \Rightarrow \quad 3^{2x} = 3^3 \quad \Rightarrow \quad x = \dfrac{3}{2}$

$t = \dfrac{1}{3} \quad \Rightarrow \quad 3^{2x} = \dfrac{1}{3} \quad \Rightarrow \quad 3^{2x} = 3^{-1} \quad \Rightarrow \quad x = -\dfrac{1}{2}$

2. $\dfrac{2^x - 16}{27 - 3^x} \geq 0$

$N \geq 0: \; 2^x - 16 \geq 0 \quad \Rightarrow \quad 2^x \geq 2^4 \quad \Rightarrow \quad x \geq 4$

$D > 0: \; 27 - 3^x > 0 \quad \Rightarrow \quad 3^x < 3^3 \quad \Rightarrow \quad x < 3$

Dallo studio dei segni ottengo: $3 < x \leq 4$

3. $4^{3x+2} - 65 \cdot 8^x \geq -4$

$4^2 \cdot 4^{3x} - 65 \cdot 8^x + 4 \geq 0$

$4^2 \cdot 2^{6x} - 65 \cdot 2^{3x} + 4 \geq 0$ Pongo: $2^{3x} = t$

$16t^2 - 65t + 4 \geq 0$ $t_{1,2} = \dfrac{65 \pm \sqrt{65^2 - 256}}{32} \left\langle \begin{array}{l} 4 \\ \dfrac{1}{16} \end{array} \right.$

$t \leq \dfrac{1}{16} \quad \vee \quad t \geq 4$

$2^{3x} \leq \dfrac{1}{16} \quad \vee \quad 2^{3x} \geq 4 \quad \Rightarrow \quad 3x \leq -4 \quad \vee \quad 3x \geq 2 \quad \Rightarrow \quad x \leq -\dfrac{4}{3} \quad \vee \quad x \geq \dfrac{2}{3}$

4. $\log_5(x-3)+\log_5(x+1)=3+\log_5\frac{12}{125}$

$c.a.:\begin{cases}x-3>0\\x+1>0\end{cases}$ $\begin{cases}x>3\\x>-1\end{cases}$ $c.a.: x>3$

$\log_5(x-3)+\log_5(x+1)=\log_5 125+\log_5\frac{12}{125}$

$\log_5(x-3)(x+1)=\log_5 125\cdot\frac{12}{125}$ $\log_5(x^2-2x-3)=\log_5 12$

$x^2-2x-3=12$ $x^2-2x-15=0$

$x_{1,2}=\frac{1\pm\sqrt{1+15}}{1}=\left\langle\begin{array}{l}5\\-3 \quad non\ acc.\ per\ c.a.\end{array}\right.$ $\Rightarrow$ $x=5$

5. $\log_{\frac{1}{4}}(5x+4)>-1$

$c.a.: 5x+4>0$ $\Rightarrow$ $x>-\frac{4}{5}$

$5x+4<4$ $5x<0$ $x<0$

Mettendo a sistema il risultato ottenuto con le c.a. ottengo: $-\frac{4}{5}<x<0$

6. $\log(16x-3)-2\log(2x+3)+\log\left(\frac{x}{4}-1\right)\geq 0$

$c.a.:\begin{cases}16x-3>0\\2x+3>0\\\frac{x}{4}-1>0\end{cases}$ $\begin{cases}x>\frac{3}{16}\\x>-\frac{3}{2}\\x>4\end{cases}$ $c.a.: x>4$

$\log(16x-3)+\log\left(\frac{x}{4}-1\right)\geq 2\log(2x+3)$

$\log(16x-3)\left(\frac{x}{4}-1\right)\geq\log(2x+3)^2$ $4x^2-16x-\frac{3x}{4}+3\geq 4x^2+12x+9$

$-\frac{115}{4}x\geq 6$ $\frac{115}{4}x\leq -6$ $x\leq-\frac{24}{115}$

Mettendo a sistema il risultato ottenuto con le c.a. ottengo: $imp.$