

$$1. \frac{a^2}{a^2-1} + \frac{1-a}{2a+2} + \frac{a+1}{2a-2} + \frac{4a-1}{1-a^2}$$

$$\begin{aligned}
 &= \frac{a^2}{(a-1)(a+1)} + \frac{1-a}{2(a+1)} + \frac{a+1}{2(a-1)} + \frac{4a-1}{-(a-1)(a+1)} = \\
 &= \frac{2a^2 + (1-a)(a-1) + (a+1)^2 - 2(4a-1)}{2(a-1)(a+1)} = \frac{2a^2 + a - 1 - a^2 + a + a^2 + 2a + 1 - 8a + 2}{2(a-1)(a+1)} = \\
 &= \frac{2a^2 - 4a + 2}{2(a-1)(a+1)} = \frac{2(a^2 - 2a + 1)}{2(a-1)(a+1)} = \frac{(a-1)^2}{(a-1)(a+1)} = \frac{\cancel{a-1}}{\cancel{a+1}}
 \end{aligned}$$

$$2. \frac{b^3-8}{9-b^2} \cdot \frac{2b^2+5b-3}{b^2+2b+4} \cdot \frac{b^2-6b+9}{4b^2-10b+4}$$

$$\begin{aligned}
 &= \frac{(b-2)(b^2+2b+4)}{(3-b)(3+b)} \cdot \frac{2b^2+6b-b-3}{b^2+2b+4} \cdot \frac{(b-3)^2}{2(2b^2-5b+2)} = \\
 &= \frac{b-2}{-(b-3)(b+3)} \cdot (2b(b+3) - 1(b+3)) \cdot \frac{(b-3)^2}{2(2b^2-4b-b+2)} = \\
 &= -\frac{b-2}{b+3} \cdot (b+3)(2b-1) \cdot \frac{b-3}{2(2b(b-2)-1(b-2))} = \\
 &= -(b-2)(2b-1) \cdot \frac{b-3}{2(b-2)(2b-1)} = \frac{\cancel{3-b}}{2}
 \end{aligned}$$

$$3. \left(\frac{x+4}{x-2} + \frac{1}{x^2-4x+4} \right) \cdot \frac{x^2+x-6}{x^3+2x^2-7x}$$

$$= \left(\frac{x+4}{x-2} + \frac{1}{(x-2)^2} \right) \cdot \frac{(x+3)(x-2)}{x(x^2+2x-7)} = \quad C.E.: \begin{cases} x \neq 2 \\ x \neq 0 \\ x^2+2x-7 \neq 0 \end{cases}$$

$$= \frac{x^2-2x+4x-8+1}{(x-2)^2} \cdot \frac{(x+3)(x-2)}{x(x^2+2x-7)} = \frac{x^2+2x-7}{(x-2)^2} \cdot \frac{(x+3)(x-2)}{x(x^2+2x-7)} = \frac{\cancel{x+3}}{\cancel{x(x-2)}}$$

4. $\left(\frac{2}{3x} - \frac{x+1}{x^2} + \frac{1}{2x} \right) : \frac{x^2-2x-24}{5x^2+20x}$

$$= \frac{4x - 6x - 6 + 3x}{6x^2} : \frac{(x-6)(x+4)}{5x(x+4)} = C.E.: \begin{cases} x \neq 0 \\ x \neq 6 \\ x \neq -4 \end{cases}$$

$$= \frac{x-6}{6x^2} \cdot \frac{5x(x+4)}{(x-6)(x+4)} = \frac{5}{6x}$$

5. $\left(\frac{a+b}{a-b} - 1 \right)^{-2} : \left(\frac{2ab}{a^2-b^2} \right)^{-1}$

$$= \left(\frac{a+b-a+b}{a-b} \right)^{-2} : \left(\frac{2ab}{(a-b)(a+b)} \right)^{-1} C.E.: \begin{cases} a \neq \pm b \\ a \neq 0 \\ b \neq 0 \end{cases}$$

$$= \left(\frac{2b}{2b} \right)^2 : \frac{(a-b)(a+b)}{2ab} =$$

$$= \frac{(a-b)^2}{4b^2} \cdot \frac{2ab}{(a-b)(a+b)} = \frac{\mathbf{a(a-b)}}{\mathbf{2b(a+b)}}$$