

1.  $4 \operatorname{sen}^2 x \cos^2 x = 1$

$$(2 \operatorname{sen} x \cos x)^2 = 1$$

$$2x = \frac{\pi}{2} + k\pi$$

$$\operatorname{sen}^2 2x = 1$$

$$x = \frac{\pi}{4} + k \frac{\pi}{2}$$

$$\operatorname{sen} 2x = \pm 1$$

2.  $\cos x - \operatorname{ctg} x = 0$

$$\cos x - \frac{\cos x}{\operatorname{sen} x} = 0$$

$$\operatorname{ctg} x = 0$$

$$\operatorname{sen} x = 1$$

$$\frac{\cos x}{\operatorname{sen} x} (\operatorname{sen} x - 1) = 0$$

$$x = \frac{\pi}{2} + k\pi$$

$$x = \frac{\pi}{2} + 2k\pi$$

$$\operatorname{ctg} x (\operatorname{sen} x - 1) = 0$$

$$x = \frac{\pi}{2} + k\pi$$

3.  $3 \operatorname{sen} x + 2 (\operatorname{sen}^2 x + 2 \cos^2 x) = 2$

$$3 \operatorname{sen} x + 2 (\operatorname{sen}^2 x + 2 (1 - \operatorname{sen}^2 x)) = 2$$

$$3 \operatorname{sen} x + 2 (\operatorname{sen}^2 x + 2 - 2 \operatorname{sen}^2 x) = 2$$

$$3 \operatorname{sen} x + 2 (2 - \operatorname{sen}^2 x) = 2$$

$$3 \operatorname{sen} x + 4 - 2 \operatorname{sen}^2 x = 2$$

$$2 \operatorname{sen}^2 x - 3 \operatorname{sen} x - 2 = 0$$

$$\operatorname{sen} x = \frac{3 \pm \sqrt{9 + 16}}{4} = \left\langle \begin{array}{l} 2 \\ -\frac{1}{2} \end{array} \right.$$

$$\operatorname{sen} x = 2$$

*imp.*

$$\operatorname{sen} x = -\frac{1}{2}$$

$$x = \frac{7}{6}\pi + 2k\pi$$

$$x = \frac{11}{6}\pi + 2k\pi$$

4.  $\cos x (2 \cos x + 1) = 1$

$$2 \cos^2 x + \cos x - 1 = 0$$

$$\cos x = \frac{1}{2}$$

$$\cos x = -1$$

$$\cos x = \frac{-1 \pm \sqrt{1 + 8}}{4} = \left\langle \begin{array}{l} \frac{1}{2} \\ -1 \end{array} \right.$$

$$x = \frac{\pi}{3} + 2k\pi$$

$$x = \frac{5}{3}\pi + 2k\pi$$

$$x = \pi + 2k\pi$$

5.  $(2 \operatorname{ctg} x + \sqrt{3})^2 = 3$

$$2 \operatorname{ctg} x + \sqrt{3} = \pm \sqrt{3}$$

$$2 \operatorname{ctg} x + \sqrt{3} = \sqrt{3}$$

$$\operatorname{ctg} x = 0$$

$$x = \frac{\pi}{2} + k\pi$$

$$2 \operatorname{ctg} x + \sqrt{3} = -\sqrt{3}$$

$$\operatorname{ctg} x = -\sqrt{3}$$

$$x = \frac{5}{6}\pi + k\pi$$

6.  $\cos\left(x - \frac{2}{3}\pi\right) = \cos(\pi + x)$

$$\cos x \cos \frac{2}{3}\pi + \operatorname{sen} x \operatorname{sen} \frac{2}{3}\pi = -\cos x$$

$$-\frac{1}{2}\cos x + \frac{\sqrt{3}}{2}\operatorname{sen} x = -\cos x$$

$$\cos x + \sqrt{3}\operatorname{sen} x = 0$$

È un'equazione omogenea di primo grado, divido per  $\cos x$ , sapendo che  $\cos x = 0$  non è soluzione dell'equazione:

$$1 + \sqrt{3}\operatorname{tg} x = 0$$

$$\operatorname{tg} x = -\frac{\sqrt{3}}{3}$$

$$x = -\frac{\pi}{6} + k\pi$$

7.  $\frac{\cos x}{1 + \operatorname{sen} x} = \sqrt{3}$

*c.a.*:  $\operatorname{sen} x \neq -1$

$$\cos x = \sqrt{3}(1 + \operatorname{sen} x)$$

$$\cos x - \sqrt{3} - \sqrt{3}\operatorname{sen} x = 0$$

Si tratta di un'equazione lineare, che risolvo in modo grafico, ponendo  $\cos x = X$  e  $\operatorname{sen} x = Y$ :

$$\begin{cases} X - \sqrt{3}Y = \sqrt{3} \\ X^2 + Y^2 = 1 \end{cases}$$

$$\begin{cases} X = \sqrt{3}Y + \sqrt{3} \\ 3Y^2 + 3 + 6Y + Y^2 = 1 \end{cases}$$

$$\begin{cases} X = -\sqrt{3}Y + \sqrt{3} \\ 2Y^2 + 3Y + 1 = 0 \end{cases}$$

$$Y = \frac{-3 \pm \sqrt{9-8}}{4} = \begin{cases} -1 & \text{non acc. per c.a.} \\ -\frac{1}{2} \end{cases}$$

$$\begin{cases} X = -\frac{\sqrt{3}}{2} + \sqrt{3} \\ Y = -\frac{1}{2} \end{cases}$$

$$\begin{cases} X = \frac{\sqrt{3}}{2} \\ Y = -\frac{1}{2} \end{cases}$$

$$x = \frac{11}{6}\pi + 2k\pi$$

$$8. \quad 5 \operatorname{sen}^2 x + (1 - \sqrt{3}) \operatorname{sen} x \cos x + (4 - \sqrt{3}) \cos^2 x = 4$$

È un'equazione di secondo grado riconducibile a omogenea, moltiplicando 4 per  $\cos^2 x + \operatorname{sen}^2 x$ :

$$5 \operatorname{sen}^2 x + (1 - \sqrt{3}) \operatorname{sen} x \cos x + 4 \cos^2 x - \sqrt{3} \cos^2 x = 4 (\cos^2 x + \operatorname{sen}^2 x)$$

$$5 \operatorname{sen}^2 x + (1 - \sqrt{3}) \operatorname{sen} x \cos x + 4 \cos^2 x - \sqrt{3} \cos^2 x = 4 \cos^2 x + 4 \operatorname{sen}^2 x$$

$$\operatorname{sen}^2 x + (1 - \sqrt{3}) \operatorname{sen} x \cos x - \sqrt{3} \cos^2 x = 0$$

Divido per  $\cos^2 x$ , avendo verificato che  $x = \frac{\pi}{2} + k\pi$  non è soluzione dell'equazione

$$\operatorname{tg}^2 x + (1 - \sqrt{3}) \operatorname{tg} x - \sqrt{3} = 0$$

$$\operatorname{tg} x = \frac{\sqrt{3} - 1 \pm \sqrt{1 + 3 - 2\sqrt{3} + 4\sqrt{3}}}{2} = \frac{\sqrt{3} - 1 \pm (1 + \sqrt{3})}{2} = \begin{cases} -1 \\ \sqrt{3} \end{cases}$$

$$\operatorname{tg} x = -1$$

$$x = \frac{3}{4} \pi + k\pi$$

$$\operatorname{tg} x = \sqrt{3}$$

$$x = \frac{\pi}{3} + k\pi$$

$$9. \quad \operatorname{sen} 3x - \cos 3x = 1$$

È un'equazione lineare che posso risolvere in forma grafica, ponendo  $\cos 3x = X$  e  $\operatorname{sen} 3x = Y$ :

$$\begin{cases} Y - X = 1 \\ X^2 + Y^2 = 1 \end{cases}$$

$$\begin{cases} X = Y - 1 \\ Y^2 - 2Y + 1 + Y^2 = 1 \end{cases}$$

$$\begin{cases} X = Y - 1 \\ 2Y^2 - 2Y = 0 \end{cases}$$

$$2Y(Y - 1) = 0 \quad \begin{cases} Y = 0 \\ Y = 1 \end{cases}$$

$$\begin{cases} X = -1 \\ Y = 0 \end{cases}$$

$$3x = \pi + 2k\pi$$

$$x = \frac{\pi}{3} + \frac{2}{3}k\pi$$

$$\begin{cases} X = 0 \\ Y = 1 \end{cases}$$

$$3x = \frac{\pi}{2} + 2k\pi$$

$$x = \frac{\pi}{6} + \frac{2}{3}k\pi$$