1. $3\,tg^3\,x - 3\,tg^2\,x - tg\,x + 1 = 0$

$3\,tg^2\,x\,(tg\,x - 1) - 1\,(tg\,x - 1) = 0 \qquad (tg\,x - 1)\,(3\,tg^2\,x - 1) = 0$

$tg\,x = 1 \qquad \boxed{x = \frac{\pi}{4} + k\pi}$

$tg\,x = \pm\frac{\sqrt{3}}{3} \qquad \boxed{x = \pm\frac{\pi}{6} + k\pi}$

2. $2\cos^2 x + (2 + \sqrt{3})\cos x + \sqrt{3} = 0$

$$\cos x = \frac{-2 - \sqrt{3} \pm \sqrt{4 + 3 + 4\sqrt{3} - 8\sqrt{3}}}{4} = \frac{-2 - \sqrt{3} \pm (2 - \sqrt{3})}{4} = \left\langle \begin{array}{l} -1 \\ -\frac{\sqrt{3}}{2} \end{array} \right.$$

$\cos x = -1 \qquad \boxed{x = \pi + 2k\pi}$

$\cos x = -\frac{\sqrt{3}}{2} \qquad \boxed{x = \frac{5}{6}\pi + 2k\pi} \qquad \boxed{x = \frac{7}{6}\pi + 2k\pi}$

3. $tg\left(\frac{\pi}{4} - x\right) + tg\left(\frac{\pi}{4} + x\right) = 4$

$\frac{\pi}{4} - x + \frac{\pi}{4} + x = \frac{\pi}{2} \quad \Rightarrow \quad \frac{\pi}{4} + x = \frac{\pi}{2} - \left(\frac{\pi}{4} - x\right)$

$tg\left(\frac{\pi}{4} - x\right) + tg\left(\frac{\pi}{2} - \left(\frac{\pi}{4} - x\right)\right) = 4 \qquad tg\left(\frac{\pi}{4} - x\right) + ctg\left(\frac{\pi}{4} - x\right) = 4$

$tg\left(\frac{\pi}{4} - x\right) + \frac{1}{tg\left(\frac{\pi}{4} - x\right)} = 4 \qquad tg\left(\frac{\pi}{4} - x\right) \neq 0$

$tg^2\left(\frac{\pi}{4} - x\right) - 4\,tg\left(\frac{\pi}{4} - x\right) + 1 = 0 \qquad tg\left(\frac{\pi}{4} - x\right) = \frac{2 \pm \sqrt{4 - 1}}{1} = \left\langle \begin{array}{l} 2 + \sqrt{3} \\ 2 - \sqrt{3} \end{array} \right.$

$tg\left(\frac{\pi}{4} - x\right) = 2 + \sqrt{3} \qquad \frac{\pi}{4} - x = \frac{5}{12}\pi + k\pi \qquad \boxed{x = -\frac{\pi}{6} + k\pi}$

$tg\left(\frac{\pi}{4} - x\right) = 2 - \sqrt{3} \qquad \frac{\pi}{4} - x = \frac{\pi}{12} + k\pi \qquad \boxed{x = \frac{\pi}{6} + k\pi}$

4. $\cos^2 \frac{x}{2} - 2\, sen^2 \frac{x}{2} = \frac{\cos 2x - 1}{4}$

$\frac{1 + \cos x}{2} - 2\,\frac{1 - \cos x}{2} = \frac{-\,sen^2 x}{2}$ $\qquad 1 + \cos x - 2 + 2\cos x = -1 + \cos^2 x$

$\cos x\,(\cos x - 3) = 0$ $\qquad \cos x = 0$ $\qquad \boxed{x = \frac{\pi}{2} + k\pi}$

5. $sen\left(\frac{\pi}{4} + \frac{x}{3}\right) + \cos\left(\frac{\pi}{4} + \frac{x}{3}\right) - 1 = 0$

È un'equazione lineare, che risolvo in modo grafico, ponendo: $sen\left(\frac{\pi}{4} + \frac{x}{3}\right) = Y$ e $\cos\left(\frac{\pi}{4} + \frac{x}{3}\right) = X$:

$\begin{cases} X + Y - 1 = 0 \\ X^2 + Y^2 = 1 \end{cases}$ $\qquad \begin{cases} X = 1 - Y \\ Y^2 + 1 - 2Y + Y^2 = 1 \end{cases}$ $\qquad \begin{cases} X = 1 - Y \\ 2Y^2 - 2Y = 0 \end{cases}$

$2\,Y\,(Y - 1) = 0 \quad \Rightarrow \quad \begin{array}{l} 1 \\ 0 \end{array}$ $\qquad \begin{cases} X = 0 \\ Y = 1 \end{cases}$ $\quad \begin{cases} X = 1 \\ Y = 0 \end{cases}$

$\frac{\pi}{4} + \frac{x}{3} = \frac{\pi}{2} + 2k\pi$ $\qquad \frac{x}{3} = \frac{\pi}{4} + 2k\pi$ $\qquad \boxed{x = \frac{3}{4}\pi + 6k\pi}$

$\frac{\pi}{4} + \frac{x}{3} = 2k\pi$ $\qquad \frac{x}{3} = -\frac{\pi}{4} + 2k\pi$ $\qquad \boxed{x = -\frac{3}{4}\pi + 6k\pi}$

6. $3\, sen^2 x - \cos^2 x = 0$

È un'equazione omogenea di secondo grado: divido per $\cos^2 x$, avendo verificato che $x = \frac{\pi}{2} + k\pi$ non è soluzione dell'equazione:

$3\, tg^2 x - 1 = 0$ $\qquad tg\ x = \pm \frac{\sqrt{3}}{3}$ $\qquad \boxed{x = \pm \frac{\pi}{6} + k\pi}$

7. $5\cos^2 x - \sqrt{3}\, sen\, 2x - sen^2 x = 2$

È un'equazione di secondo grado riconducibile a omogenea, moltiplicando 2 per $\cos^2 x + sen^2 x$:

$5\cos^2 x - \sqrt{3}\, sen\, 2x - sen^2 x = 2\,(\cos^2 x + sen^2 x)$

$5\cos^2 x - \sqrt{3}\, sen\, 2x - sen^2 x = 2\cos^2 x + 2\, sen^2 x$

$3\, sen^2 x + 2\sqrt{3}\, sen\, x \cos x - 3\cos^2 x = 0$

Divido per $\cos^2 x$, avendo verificato che $x = \frac{\pi}{2} + k\pi$ non è soluzione dell'equazione

$3\, tg^2 x + 2\sqrt{3}\, tg\, x - 3 = 0$

$$tg\, x = \frac{-\sqrt{3} \pm \sqrt{3+9}}{3} = \frac{-\sqrt{3} \pm 2\sqrt{3}}{3} = \left\langle \begin{array}{l} -\sqrt{3} \\ \frac{\sqrt{3}}{3} \end{array} \right.$$

$tg\, x = -\sqrt{3}$ $\quad x = \frac{2}{3}\pi + k\pi$

$tg\, x = \frac{\sqrt{3}}{3}$ $\quad x = \frac{\pi}{6} + k\pi$

$$\boxed{x = \frac{\pi}{6} + k\frac{\pi}{2}}$$

8. $sen\, 4x - \cos 4x = 1$

È un'equazione lineare che posso risolvere in forma grafica, ponendo cos 4x = X e sen 4x = Y:

$\begin{cases} Y - X = 1 \\ X^2 + Y^2 = 1 \end{cases}$ $\quad \begin{cases} X = Y - 1 \\ Y^2 - 2Y + 1 + Y^2 = 1 \end{cases}$ $\quad \begin{cases} X = Y - 1 \\ 2Y^2 - 2Y = 0 \end{cases}$

$2Y\,(Y-1) = 0$ $\quad \left\langle \begin{array}{l} Y = 0 \\ Y = 1 \end{array} \right.$

$\begin{cases} X = -1 \\ Y = 0 \end{cases}$ $\quad 4x = \pi + 2k\pi$ $\quad \boxed{x = \frac{\pi}{4} + k\frac{\pi}{2}}$

$\begin{cases} X = 0 \\ Y = 1 \end{cases}$ $\quad 4x = \frac{\pi}{2} + 2k\pi$ $\quad \boxed{x = \frac{\pi}{8} + k\frac{\pi}{2}}$