1. $\dfrac{6-2\sqrt{5}}{\sqrt{5}-2}\cdot\dfrac{\sqrt{5}}{3-\sqrt{5}}\cdot\dfrac{10}{5+\sqrt{5}}$

$=\dfrac{2\left(3-\sqrt{5}\right)}{\sqrt{5}-2}\cdot\dfrac{\sqrt{5}}{3-\sqrt{5}}\cdot\dfrac{10}{\sqrt{5}\left(\sqrt{5}+1\right)}=\dfrac{20}{5+\sqrt{5}-2\sqrt{5}-2}=\dfrac{20}{3-\sqrt{5}}\cdot\dfrac{3+\sqrt{5}}{3+\sqrt{5}}=\mathbf{5\left(3+\sqrt{5}\right)}$

2. $\dfrac{\left(\sqrt{6}-2\sqrt{2}\right)\sqrt{\sqrt{5}-1}}{\sqrt{\sqrt{5}+1}\left(\sqrt{3}-2\right)\left(\sqrt{5}-1\right)}+\dfrac{1}{\sqrt{2}}+\dfrac{1}{\sqrt{2}-1}$

$=\dfrac{\sqrt{2}\left(\sqrt{3}-2\right)\sqrt{\sqrt{5}-1}}{\sqrt{\sqrt{5}+1}\left(\sqrt{3}-2\right)\left(\sqrt{5}-1\right)}\cdot\dfrac{\sqrt{\sqrt{5}-1}}{\sqrt{\sqrt{5}-1}}+\dfrac{1}{\sqrt{2}}\cdot\dfrac{\sqrt{2}}{\sqrt{2}}+\dfrac{1}{\sqrt{2}-1}\cdot\dfrac{\sqrt{2}+1}{\sqrt{2}+1}=$

$=\dfrac{\sqrt{2}\left(\sqrt{5}-1\right)}{\sqrt{5}-1\left(\sqrt{5}-1\right)}+\dfrac{\sqrt{2}}{2}+\sqrt{2}+1=\dfrac{\sqrt{2}}{2}+\dfrac{\sqrt{2}}{2}+\sqrt{2}+1=\mathbf{2\sqrt{2}+1}$

3. $\dfrac{-\sqrt{8-2\sqrt{7}}\left(1+\sqrt{7}\right)}{\sqrt{6}}+\left(\sqrt{2}+\sqrt{3}\right)^{2}-\sqrt{3}\left(\sqrt{2}+6\right)+6\sqrt{3}$

$=\dfrac{-\sqrt{\left(\sqrt{7}-1\right)^{2}\left(1+\sqrt{7}\right)}}{\sqrt{6}}\cdot\dfrac{\sqrt{6}}{\sqrt{6}}+2+3+2\sqrt{6}-\sqrt{6}-6\sqrt{3}+6\sqrt{3}=\dfrac{\sqrt{6}\left(1-\sqrt{7}\right)\left(1+\sqrt{7}\right)}{6}+5+\sqrt{6}=-\sqrt{6}+5+\sqrt{6}=\mathbf{5}$

4. $\left[2\sqrt{2}+\sqrt{15}:\sqrt{\left(2\sqrt{3}-3\right)\left(2\sqrt{3}+3\right)}\right]^{2}-\sqrt{6}\left(\sqrt{24}+\sqrt{15}\right)$

$=\left[2\sqrt{2}+\dfrac{\sqrt{15}}{\sqrt{12-9}}\right]^{2}-12-3\sqrt{10}=\left(2\sqrt{2}+\sqrt{5}\right)^{2}-12-3\sqrt{10}=8+5+4\sqrt{10}-12-3\sqrt{10}=\mathbf{1+\sqrt{10}}$

5. $\dfrac{\sqrt{6}+3\sqrt{2}+\sqrt{15}+3\sqrt{5}}{\sqrt{2}+\sqrt{5}-2-\sqrt{10}}\cdot\dfrac{1}{\sqrt{3}+1}$

$=\dfrac{\sqrt{2}\left(\sqrt{3}+3\right)+\sqrt{5}\left(\sqrt{3}+3\right)}{\sqrt{2}\left(1-\sqrt{2}\right)+\sqrt{5}\left(1-\sqrt{2}\right)}\cdot\dfrac{1}{\sqrt{3}+1}=\dfrac{\left(\sqrt{2}+\sqrt{5}\right)\left(\sqrt{3}+3\right)}{\left(\sqrt{2}+\sqrt{5}\right)\left(1-\sqrt{2}\right)}\cdot\dfrac{1}{\sqrt{3}+1}=\dfrac{\sqrt{3}\left(1+\sqrt{3}\right)}{1-\sqrt{2}}\cdot\dfrac{1}{\sqrt{3}+1}\cdot\dfrac{1+\sqrt{2}}{1+\sqrt{2}}=\mathbf{-\sqrt{3}-\sqrt{6}}$

6. $\left(\dfrac{\sqrt{2}+1}{1-\sqrt{2}}-\dfrac{2}{\sqrt{2}+1}\right)\left(\dfrac{\sqrt{2}+3}{\sqrt{2}}-\sqrt{2}\right):\dfrac{9+5\sqrt{2}}{8}$

$=\dfrac{2+1+2\sqrt{2}-2+2\sqrt{2}}{1-2}\cdot\dfrac{\sqrt{2}+3-2}{\sqrt{2}}\cdot\dfrac{8}{9+5\sqrt{2}}=\dfrac{-\left(1+4\sqrt{2}\right)\left(\sqrt{2}+1\right)}{\sqrt{2}}\cdot\dfrac{8}{9+5\sqrt{2}}=$

$=-\dfrac{\sqrt{2}+1+8+4\sqrt{2}}{\sqrt{2}}\cdot\dfrac{8}{9+5\sqrt{2}}=-\dfrac{8}{\sqrt{2}}\cdot\dfrac{\sqrt{2}}{\sqrt{2}}=\mathbf{-4\sqrt{2}}$

7. $\dfrac{\left(1-\sqrt{2}\right)^{2}}{3\sqrt{2}}-\dfrac{3}{4}\sqrt{2}+\dfrac{1}{3}\left(1+\sqrt{2}\right)\left(1-\dfrac{1}{\sqrt{2}}\right)+\dfrac{2}{3}$

$=\dfrac{1+2-2\sqrt{2}}{3\sqrt{2}}-\dfrac{3}{4}\sqrt{2}+\dfrac{\left(1+\sqrt{2}\right)\left(\sqrt{2}-1\right)}{3\sqrt{2}}+\dfrac{2}{3}=\dfrac{1}{\sqrt{2}}-\dfrac{2}{3}-\dfrac{3}{4}\sqrt{2}+\dfrac{1}{3\sqrt{2}}+\dfrac{2}{3}=\dfrac{\sqrt{2}}{2}-\dfrac{3}{4}\sqrt{2}+\dfrac{\sqrt{2}}{6}=\dfrac{\sqrt{2}\left(6-9+2\right)}{12}=\mathbf{-\dfrac{\sqrt{2}}{12}}$

8. $\left(\dfrac{\sqrt{3}}{\sqrt{2}+\sqrt{3}}-\sqrt{6}\right)\left(3+2\sqrt{6}\right)$

$=\left(\dfrac{\sqrt{3}}{\sqrt{2}+\sqrt{3}}\cdot\dfrac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}}-\sqrt{6}\right)\left(3+2\sqrt{6}\right)=\left(3-\sqrt{6}-\sqrt{6}\right)\left(3+2\sqrt{6}\right)=9-24=\mathbf{-15}$

9. $\left[3\left(\sqrt{5}+\sqrt{2}\right)^2 - 2\left(\sqrt{10}+1\right)^2 + 1\right] : \sqrt{40} + \dfrac{1}{\sqrt{5}-2}$

$= \left[3\left(5 + 2 + 2\sqrt{10}\right) - 2\left(10 + 1 + 2\sqrt{10}\right) + 1\right] : \left(2\sqrt{10}\right) + \dfrac{1}{\sqrt{5}-2} \cdot \dfrac{\sqrt{5}+2}{\sqrt{5}+2} =$

$= \left(21 + 6\sqrt{10} - 22 - 4\sqrt{10} + 1\right) : \left(2\sqrt{10}\right) + \dfrac{\sqrt{5}+2}{5-4} = 2\sqrt{10} : \left(2\sqrt{10}\right) + \sqrt{5} + 2 = 1 + \sqrt{5} + 2 = \mathbf{\sqrt{5}+3}$

10. Osserva la seguente figura.

Sapendo che quelle riportate sono le lunghezze delle basi del trapezio e della base del triangolo, che trapezio e triangolo sono equivalenti, cioè hanno la stessa area, e che l'altezza del trapezio è 6 volte il reciproco della somma delle sue basi, determina l'altezza del triangolo.

Sapendo che nel trapezio $h = \dfrac{6}{b+B}$ e che l'area si calcola come:

$$A = \dfrac{(b+B)\cdot h}{2} = \dfrac{(b+B)\cdot \dfrac{6}{b+B}}{2} = 3$$
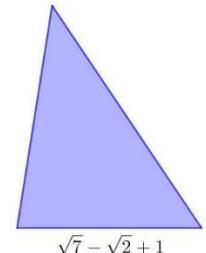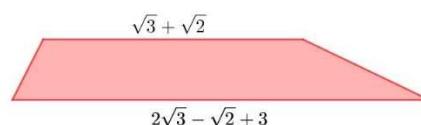
Sapendo che l'area del triangolo si calcola facendo: $A = \dfrac{b\cdot h}{2}$ e sapendo che l'area del triangolo, equivalente al trapezio, è 3, otteniamo:

$$h = \dfrac{2A}{b} = \dfrac{6}{b} = \dfrac{6}{\sqrt{7}-\sqrt{2}+1} \cdot \dfrac{\sqrt{7}-\sqrt{2}-1}{\sqrt{7}-\sqrt{2}-1} = \dfrac{6\left(\sqrt{7}-\sqrt{2}-1\right)}{7+2-2\sqrt{14}-1} =$$

$$= \dfrac{3\left(\sqrt{7}-\sqrt{2}-1\right)}{4-\sqrt{14}} \cdot \dfrac{4+\sqrt{14}}{4+\sqrt{14}} = 3 \cdot \dfrac{4\sqrt{7}+7\sqrt{2}-4\sqrt{2}-2\sqrt{7}-4-\sqrt{14}}{16-14} = \mathbf{\dfrac{3}{2}\left(2\sqrt{7}+3\sqrt{2}-\sqrt{14}-4\right)}$$

11. Stabilisci se le seguenti affermazioni sono vere o false:

	V	F		V	F
$\sqrt[3]{-7} = -\sqrt[3]{7}$	☑	☐	$\sqrt{9-1} = \sqrt{9}-1$	☐	☑
$\sqrt{-1} = \sqrt[3]{-1}$	☐	☑	$\left(-\dfrac{\sqrt[3]{3}}{3}\right)^3 = -\dfrac{1}{9}$	☑	☐
$\sqrt[5]{-0{,}03} < 0$	☑	☐	$\left(-2\sqrt[5]{-3}\right)^5 = 96$	☑	☐
$\sqrt[6]{-6} = -\sqrt[6]{6}$	☐	☑	$\sqrt[4]{(-10)^7} = \sqrt[40]{(-10)^{70}}$	☐	☑
$\sqrt[3]{-4^2} > 0$	☐	☑	$\sqrt[5]{-3^4} = \sqrt[20]{(-3)^{16}}$	☐	☑
$\sqrt{-2} \cdot \sqrt{-2} = -2$	☐	☑	$\sqrt[8]{64} \cdot \sqrt[8]{64} = \sqrt{2^3}$	☑	☐
$\sqrt[4]{(-7)^4} = -7$	☐	☑	$\sqrt[6]{12} \cdot \sqrt[6]{2} \cdot \sqrt[6]{72} = \sqrt{24}$	☐	☑
$\sqrt{\left(1-\sqrt{5}\right)^2} = 1-\sqrt{5}$	☐	☑	$\sqrt{90} : \left(\sqrt{2} : \sqrt{5}\right) = 15$	☑	☐
$\left(\sqrt[5]{-3}\right)^4 \cdot \sqrt[5]{-3} = 3$	☐	☑	$\dfrac{\sqrt{3}+3}{4+4\sqrt{3}} = \dfrac{\sqrt{3}}{4}$	☑	☐
$\sqrt{3} + \sqrt{5} = \sqrt{8}$	☐	☑	$\sqrt[4]{\dfrac{32}{81}} = \dfrac{2}{3}\sqrt[4]{2}$	☑	☐
$\sqrt{6^2 + 8^2} = 14$	☐	☑	$\sqrt{3^8 \cdot 5^6 \cdot 2} = 3^4 \cdot 5^3 \sqrt{2}$	☑	☐
$\left(\sqrt{2}+1\right)^2 = 3$	☐	☑	$\sqrt[3]{2\sqrt{2}} = 2$	☐	☑