

Svolgimento verifica A:

$$1. \lim_{x \rightarrow +\infty} \left(\frac{x}{2+x} \right)^{-x} = \lim_{x \rightarrow +\infty} \left(1 + \frac{2}{x} \right)^x = \lim_{x \rightarrow +\infty} \left[\left(1 + \frac{2}{x} \right)^{\frac{x}{2}} \right]^2 = \lim_{y \rightarrow +\infty} \left[\left(1 + \frac{1}{y} \right)^y \right]^2 = e^2$$

$$2. \lim_{x \rightarrow +\infty} \frac{e^{2x}-1}{e^{3x}+2} = \lim_{x \rightarrow +\infty} \frac{e^{2x}}{e^{3x}} = \lim_{x \rightarrow +\infty} \frac{1}{e^x} = 0$$

$$3. \lim_{x \rightarrow 0} \frac{x+\tan x}{2x+\sin x} = \lim_{x \rightarrow 0} \frac{x+x}{2x+x} = \lim_{x \rightarrow 0} \frac{2x}{3x} = \frac{2}{3}$$

$\tan x \sim x$ e $\sin x \sim x$

$$4. \lim_{x \rightarrow 0} \frac{e^{\sin 4x}-1}{\ln(1+\tan x)} = \lim_{x \rightarrow 0} \frac{\sin 4x}{\tan x} = \lim_{x \rightarrow 0} \frac{4x}{x} = 4$$

$e^{\sin 4x} - 1 \sim \sin 4x \sim 4x$ e $\ln(1 + \tan x) \sim \tan x \sim x$

$$5. \lim_{x \rightarrow 0} \frac{e^x+e^{-x}-2}{3x} = \lim_{x \rightarrow 0} \frac{e^x-1+e^{-x}-1}{3x} = \lim_{x \rightarrow 0} \frac{e^x-1-e^{-x}(e^x-1)}{3x} = \lim_{x \rightarrow 0} \frac{(e^x-1)(1-e^{-x})}{3x} = \frac{1}{3} \left(\lim_{x \rightarrow 0} \frac{e^x-1}{x} \right) \left(\lim_{x \rightarrow 0} (1-e^{-x}) \right) = 0$$

Svolgimento verifica B:

$$1. \lim_{x \rightarrow +\infty} \left(\frac{x}{x-1} \right)^{-x} = \lim_{x \rightarrow +\infty} \left(1 - \frac{1}{x} \right)^x = \lim_{x \rightarrow +\infty} \left[\left(1 + \frac{1}{-x} \right)^{-x} \right]^{-1} = \lim_{y \rightarrow -\infty} \left[\left(1 + \frac{1}{y} \right)^y \right]^{-1} = \frac{1}{e}$$

$$2. \lim_{x \rightarrow +\infty} \frac{e^{3x}+2}{e^{2x}-1} = \lim_{x \rightarrow +\infty} \frac{e^{3x}}{e^{2x}} = \lim_{x \rightarrow +\infty} e^x = +\infty$$

$$3. \lim_{x \rightarrow 0} \frac{3x+\sin x}{x+\tan x} = \lim_{x \rightarrow 0} \frac{3x+x}{x+x} = \lim_{x \rightarrow 0} \frac{4x}{2x} = 4$$

$\tan x \sim x$ e $\sin x \sim x$

$$4. \lim_{x \rightarrow 0} \frac{\ln(1+\tan x)}{e^{\sin 2x}-1} = \lim_{x \rightarrow 0} \frac{\tan x}{\sin 2x} = \lim_{x \rightarrow 0} \frac{x}{2x} = \frac{1}{2}$$

$e^{\sin 2x} - 1 \sim \sin 2x \sim 2x$ e $\ln(1 + \tan x) \sim \tan x \sim x$

$$5. \lim_{x \rightarrow 0} \frac{e^x+e^{-x}-2}{3x} = \lim_{x \rightarrow 0} \frac{e^x-1+e^{-x}-1}{3x} = \lim_{x \rightarrow 0} \frac{e^x-1-e^{-x}(e^x-1)}{3x} = \lim_{x \rightarrow 0} \frac{(e^x-1)(1-e^{-x})}{3x} = \frac{1}{3} \left(\lim_{x \rightarrow 0} \frac{e^x-1}{x} \right) \left(\lim_{x \rightarrow 0} (1-e^{-x}) \right) = 0$$