1. Applicando la definizione di derivata calcola la derivata, in un punto generico, della funzione $y=\frac{x+1}{x-2}$.

$$\lim_{h\to 0}\frac{\frac{x+h+1}{x+h-2}-\frac{x+1}{x-2}}{h}=\lim_{h\to 0}\frac{x^2+hx+x-2x-2h-2-x^2-xh+2x-x-h+2}{h\,(x-2)(x+h-2)}=$$

$$=\lim_{h\to 0}\frac{-3h}{h\,(x-2)(x+h-2)}=\lim_{h\to 0}\frac{-3}{(x-2)(x+h-2)}=-\frac{3}{(x-2)^2}$$

Calcola le derivate delle seguenti funzioni:

2. $y=5x^2+9x^4+\frac{1}{7}x^7-\frac{3}{14}x^{21}$

$$y'=10x+36x^3+x^6-\frac{9}{2}x^{20}$$

3. $y=\sqrt[7]{x^4}-\frac{1}{\sqrt[5]{x^3}}+x\sqrt{x}-\sqrt{\sqrt[4]{x^3}}$

$$y=x^{\frac{4}{7}}-x^{-\frac{3}{5}}+x^{\frac{3}{2}}-x^{\frac{3}{8}} \qquad y'=\frac{4}{7}x^{-\frac{3}{7}}+\frac{3}{5}x^{-\frac{8}{5}}+\frac{3}{2}x^{\frac{1}{2}}-\frac{3}{8}x^{-\frac{5}{8}}=\frac{4}{7\sqrt[7]{x^3}}+\frac{3}{5\sqrt[5]{x^8}}+\frac{3}{2}\sqrt{x}-\frac{3}{8\sqrt[8]{x^5}}$$

4. $y=\frac{x^5-x\sqrt{x}}{\sqrt[3]{x}}$

$$y=x^5:x^{\frac{1}{3}}-x\cdot x^{\frac{1}{2}}:x^{\frac{1}{3}}=x^{\frac{14}{3}}-x^{\frac{7}{6}} \qquad y'=\frac{14}{3}x^{\frac{11}{3}}-\frac{7}{6}x^{\frac{1}{6}}=\frac{14}{3}x^3\sqrt[3]{x^2}-\frac{7}{6}\sqrt[6]{x}$$

5. $y=3^x-5^{x+2}$

$$y=3^x-25\cdot 5^x \qquad y'=3^x\ln 3-25\cdot 5^x\ln 5=3^x\ln 3-5^{x+2}\ln 5$$

6. $y=\log_5 x+x^4\log_3 5$

$$y'=\frac{1}{x}\log_5 e+4x^3\log_3 5$$

7. $y=\ln^2 x\cdot\log_x 4$

$$y=\ln^2 x\cdot\frac{\ln 4}{\ln x}=\ln 4\ln x \qquad y'=\frac{\ln 4}{x}$$

8. $y=\cos x-3\sin x$

$$y'=-\sin x-3\cos x$$

9. $y=\sin 2x$

$$y=2\sin x\cos x \qquad y'=2\cos^2 x-2\sin^2 x=2\cos 2x$$