1. $$\tan\left(\arcsin\frac{2\sqrt{5}}{5}+\arccos\frac{\sqrt{10}}{10}\right)$$

$$\alpha=\arcsin\frac{2\sqrt{5}}{5} \quad\Rightarrow\quad \sin\alpha=\frac{2\sqrt{5}}{5} \qquad 0<\alpha<\frac{\pi}{2} \qquad \tan\alpha=\frac{\sin\alpha}{\cos\alpha}=\frac{\sin\alpha}{\sqrt{1-\sin^2\alpha}}=\frac{\frac{2\sqrt{5}}{5}}{\sqrt{1-\frac{4}{5}}}=2$$

$$\beta=\arccos\frac{\sqrt{10}}{10} \quad\Rightarrow\quad \cos\beta=\frac{\sqrt{10}}{10} \qquad 0<\beta<\frac{\pi}{2} \qquad \tan\beta=\frac{\sin\beta}{\cos\beta}=\frac{\sqrt{1-\cos^2\beta}}{\cos\beta}=\frac{\sqrt{1-\frac{1}{10}}}{\frac{\sqrt{10}}{10}}=3$$

$$\tan\left(\arcsin\frac{2\sqrt{5}}{5}+\arccos\frac{\sqrt{10}}{10}\right)=\tan(\alpha+\beta)=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}=\frac{2+3}{1-2\cdot 3}=\mathbf{-1}$$

2. $$\frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}\cdot\tan x-\sqrt{2}\cos\left(x-\frac{\pi}{4}\right)$$

Formule parametriche razionali: $\cos x=\frac{1-t^2}{1+t^2}$ $\quad t=\tan\frac{x}{2}$

$$=\cos x\cdot\frac{\sin x}{\cos x}-\sqrt{2}\left(\frac{\sqrt{2}}{2}\cos x+\frac{\sqrt{2}}{2}\sin x\right)=\sin x-\cos x-\sin x=\boldsymbol{-\cos x}$$

3. $$\cos^2\frac{x}{4}-\frac{1}{2}\cos\frac{x}{2}+2\cos^2\left(\frac{\pi}{2}+\frac{x}{4}\right)-\frac{3}{2}$$

$$=\frac{1+\cos\frac{x}{2}}{2}-\frac{1}{2}\cos\frac{x}{2}+2\cdot\sin^2\frac{x}{4}-\frac{3}{2}=\frac{1}{2}+\frac{1}{2}\cos\frac{x}{2}-\frac{1}{2}\cos\frac{x}{2}+2\cdot\frac{1-\cos\frac{x}{2}}{2}-\frac{3}{2}=\frac{1}{2}+1-\cos\frac{x}{2}-\frac{3}{2}=\boldsymbol{-\cos\frac{x}{2}}$$

4. $$\frac{\sin\alpha\cos x}{\sin^2(\alpha-x)+\cos^2(\alpha-x)}\cdot\frac{1}{\cos(\pi-x)\sin(-\alpha)}=\frac{1}{2\left(\cos^2\left(\frac{\pi}{4}+x\right)+\cos x\sin x\right)}$$

$$\frac{\sin\alpha\cos x}{1}\cdot\frac{1}{-\cos x\cdot(-\sin\alpha)}=\frac{1}{2\left(\cos^2\left(\frac{\frac{\pi}{2}+2x}{2}\right)+\frac{1}{2}\sin 2x\right)}$$

$$\frac{\sin\alpha\cos x}{\cos x\ \sin\alpha}=\frac{1}{2\left(\frac{1+\cos\left(\frac{\pi}{2}+2x\right)}{2}+\frac{1}{2}\sin 2x\right)}$$

$$1+\cos\left(\frac{\pi}{2}+2x\right)+\sin 2x=1 \qquad 1-\sin 2x+\sin 2x=1 \qquad \mathbf{1=1}$$

5. $$\sin 3\alpha=\sin\alpha\ \left(3-4\sin^2\alpha\right)$$

$$\sin(2\alpha+\alpha)=3\sin\alpha-4\sin\alpha\cdot\frac{1-\cos 2\alpha}{2} \qquad \sin 2\alpha\cos\alpha+\cos 2\alpha\sin\alpha=3\sin\alpha-2\sin\alpha+2\sin\alpha\cos 2\alpha$$

$$\sin 2\alpha\cos\alpha-\cos 2\alpha\sin\alpha=\sin\alpha \qquad \sin(2\alpha-\alpha)=\sin\alpha \qquad \boldsymbol{\sin\alpha=\sin\alpha}$$

6. $\cos\left(\frac{\pi}{6} - (\alpha + \beta)\right) - \sin(\alpha + \beta) = \cos\left(\frac{\pi}{6} + \alpha + \beta\right)$

$\frac{\sqrt{3}}{2}\cos(\alpha + \beta) + \frac{1}{2}\sin(\alpha + \beta) - \sin(\alpha + \beta) = \frac{\sqrt{3}}{2}\cos(\alpha + \beta) - \frac{1}{2}\sin(\alpha + \beta)$ $\quad$ $\boldsymbol{-\frac{1}{2}\sin(\alpha + \beta) = -\frac{1}{2}\sin(\alpha + \beta)}$

7. $2\sin^2\alpha = \frac{\sin^2 2\alpha}{1 + \cos 2\alpha}$

$2 \cdot \frac{1 - \cos 2\alpha}{2} \cdot (1 + \cos 2\alpha) = \sin^2 2\alpha$ $\quad$ $1 - \cos^2 2\alpha = \sin^2 2\alpha$ $\quad$ $\boldsymbol{sin^2 2\alpha = sin^2 2\alpha}$

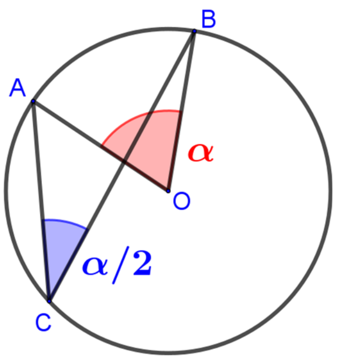
8. Sapendo che $\alpha = \arcsin\frac{1}{3}$ e $\beta = \arcsin\frac{7}{9}$, calcola $\sin(\alpha + \beta)$ e $\cos(\beta - \alpha)$.

$$\alpha = \arcsin\frac{1}{3} \Rightarrow \sin\alpha = \frac{1}{3} \quad 0 < \alpha < \frac{\pi}{2} \quad \cos\alpha = \sqrt{1 - \sin^2\alpha} = \sqrt{1 - \frac{1}{9}} = \frac{2\sqrt{2}}{3}$$

$$\beta = \arcsin\frac{7}{9} \Rightarrow \sin\beta = \frac{7}{9} \quad 0 < \beta < \frac{\pi}{2} \quad \cos\beta = \sqrt{1 - \sin^2\beta} = \sqrt{1 - \frac{49}{81}} = \frac{4\sqrt{2}}{9}$$

$$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta = \boldsymbol{\frac{2}{3}\sqrt{2}} \qquad \cos(\beta - \alpha) = \cos\alpha\cos\beta + \sin\alpha\sin\beta = \boldsymbol{\frac{23}{27}}$$

9. In una circonferenza di centro O considera una corda AB e un angolo alla circonferenza $A\hat{C}B$ tale che $\tan A\hat{C}B = \frac{3\sqrt{3}}{2}$. Calcola il coseno e il seno del corrispondente angolo al centro $A\hat{O}B$.



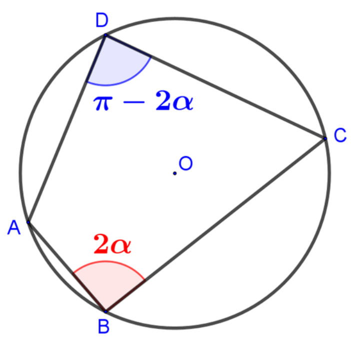
L'angolo alla circonferenza è sempre la metà dell'angolo al centro che insiste sullo stesso arco, perciò:

$$\tan\frac{\alpha}{2} = \frac{3\sqrt{3}}{2} \qquad \cos\alpha\,? \quad \sin\alpha\,?$$

Applico le formule parametriche, data $\tan\frac{\alpha}{2} = t$:

$$\cos\alpha = \frac{1 - t^2}{1 + t^2} = \frac{1 - \frac{27}{4}}{1 + \frac{27}{4}} = \boldsymbol{-\frac{23}{31}} \qquad \sin\alpha = \frac{2t}{1 + t^2} = \frac{3\sqrt{3}}{1 + \frac{27}{4}} = \boldsymbol{\frac{12\sqrt{3}}{31}}$$

10. Il quadrilatero ABCD è inscritto in una circonferenza di centro O. Sapendo che $\cos\frac{A\hat{B}C}{2} = \frac{\sqrt{5}}{4}$, determina $\cos C\hat{D}A$.



Un quadrilatero inscritto in una circonferenza ha gli angoli opposti supplementari, perciò:

$$\cos\alpha = \frac{\sqrt{5}}{4} \qquad \cos(\pi - 2\alpha)\,?$$

Applico le formule degli archi associati e quelle di duplicazione:

$$\cos(\pi - 2\alpha) = -\cos 2\alpha = -2\cos^2\alpha + 1 = \boldsymbol{\frac{3}{8}}$$

11. Data la funzione $f: x \to a \sin x + b \cos x$, determina a e b, sapendo che il grafico passa per i punti $A\left(\frac{\pi}{2}; 3\right)$ e $B(0; 4)$.

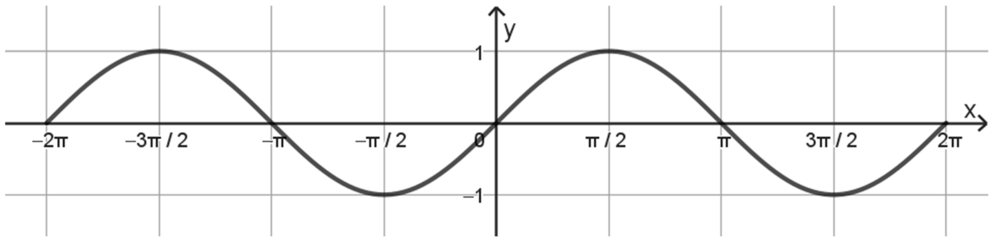
Dato che il grafico passa per i punti A e B, le coordinate dei due punti soddisferanno l'equazione, perciò:

$$\begin{cases} 3 = a \sin\frac{\pi}{2} + b \cos\frac{\pi}{2} \\ 4 = a \sin 0 + b \cos 0 \end{cases} \qquad \begin{cases} \boldsymbol{a = 3} \\ \boldsymbol{b = 4} \end{cases} \qquad y = 3 \sin x + 4 \cos x$$

12. Traccia i grafici delle seguenti funzioni nell'intervallo $[-2\pi; 2\pi]$:

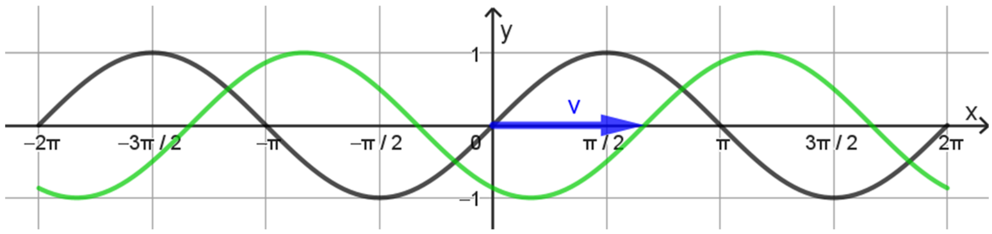
$y = 2 \sin\left(\frac{1}{2}x - \frac{\pi}{3}\right)$

$y = \sin x$



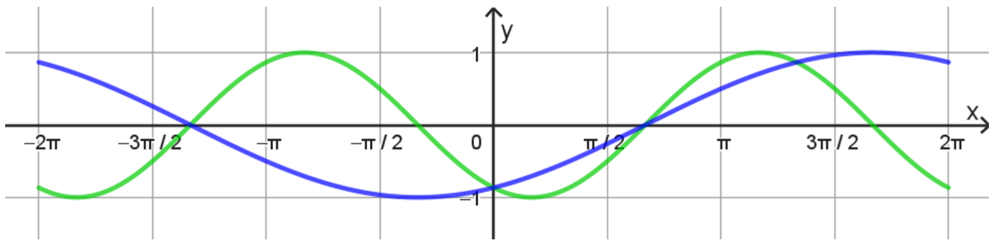
Traslazione di un vettore $\vec{v}\left(\frac{2}{3}\pi; 0\right)$

$y = \sin\left(x - \frac{2}{3}\pi\right)$



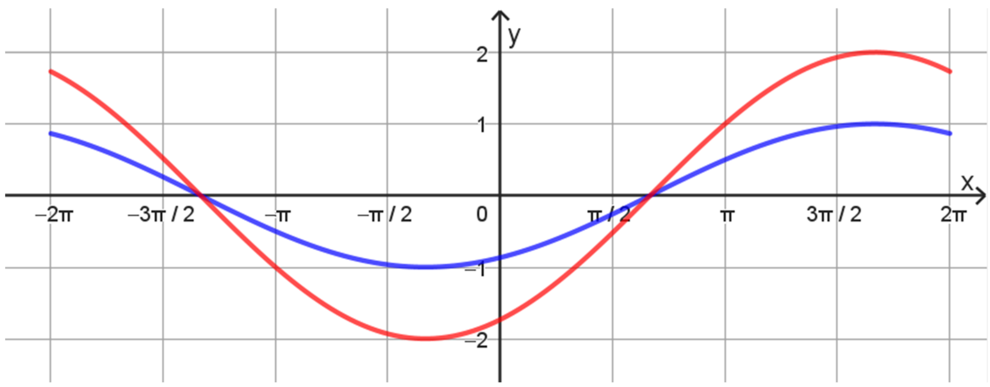
Dilatazione lungo l'asse x di un fattore 2:

$y = \sin\left(\frac{x - \frac{2}{3}\pi}{2}\right)$



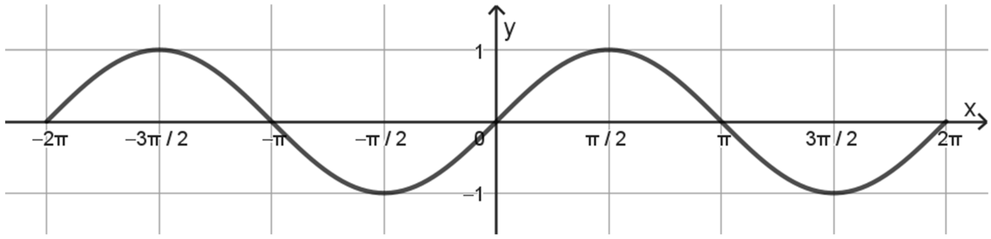
Dilatazione lungo l'asse x di un fattore 2:

$y = 2 \sin\left(\frac{1}{2}x - \frac{\pi}{3}\right)$

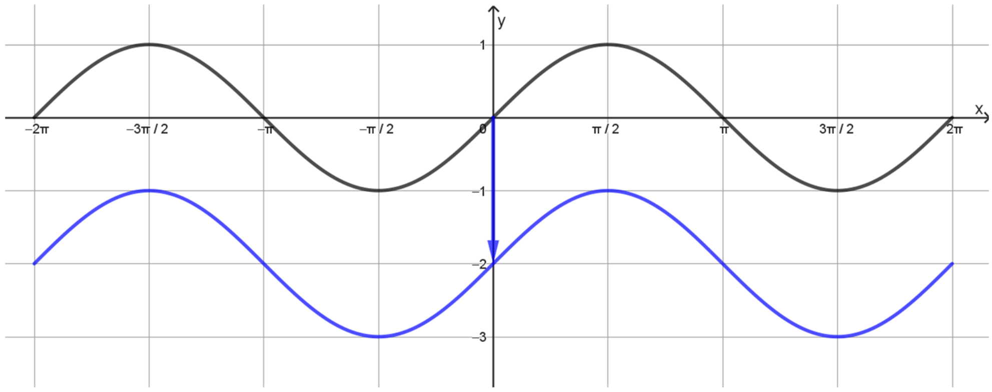


$y = |\sin x - 2|$

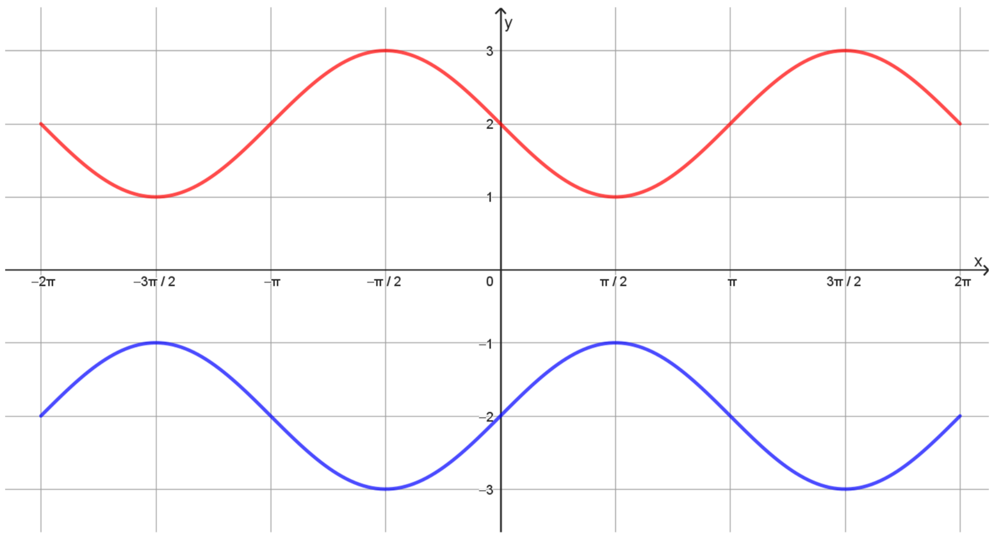
$y = \sin x$



Traslazione di un vettore
$\vec{v}\ (0; -2)$
$y = \sin x - 2$



Simmetria rispetto all'asse x:
$y = |\sin x - 2|$



13. Determina il dominio delle seguenti funzioni:

$y = \arcsin \dfrac{2x-1}{x}$

$$-1 \le \frac{2x-1}{x} \le 1 \quad \Rightarrow \quad \begin{cases} \dfrac{3x-1}{x} \ge 0 \\ \dfrac{x-1}{x} \le 0 \end{cases} \quad \Rightarrow \quad \begin{cases} x < 0 \quad \vee \quad x \ge \dfrac{1}{3} \\ 0 < x \le 1 \end{cases} \quad \Rightarrow \quad \boldsymbol{\frac{1}{3} \le x \le 1}$$

$y = \arctan \sqrt{\dfrac{x-1}{x+2}}$

$$\frac{x-1}{x+2} \ge 0 \qquad \boldsymbol{x < -2 \quad \vee \quad x \ge 1}$$