1. $\sin x + \cot x = \csc x$

$\sin x + \frac{\cos x}{\sin x} - \frac{1}{\sin x} = 0$ $\quad \frac{\sin^2 x + \cos x - 1}{\sin x} = 0$ $\quad C.A.: \sin x \neq 0$

$1 - \cos^2 x + \cos x - 1 = 0$ $\quad \cos^2 x - \cos x = 0$ $\quad \cos x\ (\cos x - 1) = 0$

$\cos x = 0$ $\quad \boldsymbol{x = \frac{\pi}{2} + k\pi}$ $\quad \cos x = 1 \quad non\ acc. per\ C.A.$

2. $\tan\left(2x - \frac{\pi}{3}\right) = -\cot\left(x - \frac{\pi}{6}\right)$

Sapendo che $\tan\left(\frac{\pi}{2} + \alpha\right) = -\cot\alpha$ ottengo: $\tan\left(2x - \frac{\pi}{3}\right) = \tan\left(\frac{\pi}{2} + x - \frac{\pi}{6}\right)$

$2x - \frac{\pi}{3} = x + \frac{\pi}{2} - \frac{\pi}{6} + k\pi$ $\quad \boldsymbol{x = \frac{2}{3}\pi + k\pi}$

3. $4\sin x \cos\left(x + \frac{11}{6}\pi\right) + 1 < \sqrt{3}\sin 2x$

$4\sin x\left(\frac{\sqrt{3}}{2}\cos x + \frac{1}{2}\sin x\right) + 1 - 2\sqrt{3}\sin x \cos x < 0$ $\quad 2\sqrt{3}\sin x \cos x + 2\sin^2 x + 1 - 2\sqrt{3}\sin x\cos x < 0$

$2\sin^2 x + 1 < 0$ $\quad \boldsymbol{\nexists x \in \mathbb{R}}$

4. $\frac{\sin 2x}{\sin x} - \sin\left(\frac{\pi}{6} - x\right) > 2\sqrt{3}\sin x$

$\frac{2\sin x\cos x}{\sin x} - \left(\frac{1}{2}\cos x - \frac{\sqrt{3}}{2}\sin x\right) > 2\sqrt{3}\sin x$ $\quad 2\cos x - \frac{1}{2}\cos x + \frac{\sqrt{3}}{2}\sin x - 2\sqrt{3}\sin x > 0$

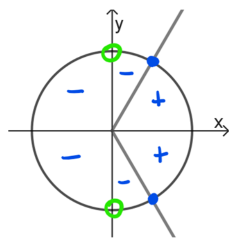
$3\cos x - 3\sqrt{3}\sin x > 0$ $\quad \sqrt{3}\sin x - \cos x < 0$ $\quad \sin\left(x - \frac{\pi}{6}\right) < 0 \quad con \quad x \neq k\pi$

$-\pi + 2k\pi < x - \frac{\pi}{6} < 0 + 2k\pi$ $\quad \boldsymbol{-\frac{5}{6}\pi + 2k\pi < x < \frac{\pi}{6} + 2k\pi \quad \wedge \quad x \neq 2k\pi}$

5. $\frac{4\cos^2\frac{x}{2} - 2 - \cos x - 2\sin^2\frac{x}{2}}{\cos^2 x - \sin^2 x + 1} \geq 0$

$N \geq 0: 4 \cdot \frac{1 + \cos x}{2} - 2 - \cos x - 2 \cdot \frac{1 - \cos x}{2} \geq 0$ $\quad 2 + 2\cos x - 2 - \cos x - 1 + \cos x \geq 0$ $\quad \cos x \geq \frac{1}{2}$

$D > 0: 2\cos^2 x > 0$ $\quad \cos x \neq 0$ $\quad x \neq \frac{\pi}{2} + k\pi$

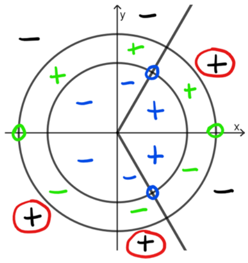


$\boldsymbol{-\frac{\pi}{3} + 2k\pi \leq x \leq \frac{\pi}{3} + 2k\pi}$

Determina il dominio delle seguenti funzioni:

6. $y = \ln\left(\dfrac{2\cos x - 1}{\sin x}\right)$

$\dfrac{2\cos x - 1}{\sin x} > 0 \qquad N > 0: \quad \cos x > \dfrac{1}{2} \qquad D > 0: \quad \sin x > 0$

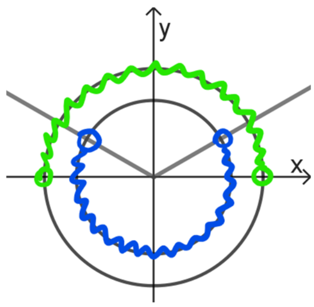


$2k\pi < x < \dfrac{\pi}{3} + 2k\pi \quad \vee$

$\pi + 2k\pi < x < \dfrac{5}{3}\pi + 2k\pi$

7. $y = \log_4\left(\log_{\frac{1}{2}} \sin x - 1\right)$

$\begin{cases} \log_{\frac{1}{2}} \sin x - 1 > 0 \\ \sin x > 0 \end{cases} \qquad \begin{cases} \sin x < \dfrac{1}{2} \\ \sin x > 0 \end{cases}$



$2k\pi < x < \dfrac{\pi}{6} + 2k\pi \quad \vee$

$\dfrac{5}{6}\pi + 2k\pi < x < \pi + 2k\pi$

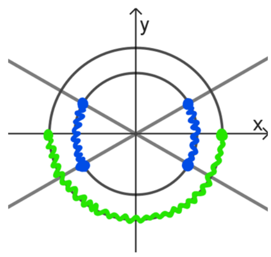
8. $y = \dfrac{1}{\sin 2x - \cos x + 2\sin x - 1}$

$2\sin x\cos x - \cos x + 2\sin x - 1 \neq 0 \qquad 2\sin x\ (\cos x + 1) - (\cos x + 1) \neq 0$

$(\cos x + 1)(2\sin x - 1) \neq 0 \qquad \cos x \neq -1 \ \wedge \ \sin x \neq \dfrac{1}{2} \qquad x \neq \pi + 2k\pi \ \wedge \ x \neq \dfrac{\pi}{6} + 2k\pi \ \wedge \ x \neq \dfrac{5}{6}\pi + 2k\pi$

9. $y = \sqrt{4\cos^2 x - 3} + \sqrt{\cos\left(\dfrac{\pi}{2} + x\right)}$

$\begin{cases} 4\cos^2 x - 3 \geq 0 \\ \cos\left(\dfrac{\pi}{2} + x\right) \geq 0 \end{cases} \qquad \begin{cases} \cos^2 x \geq \dfrac{3}{4} \\ -\sin x \geq 0 \end{cases} \qquad \begin{cases} \cos x \leq -\dfrac{\sqrt{3}}{2} \ \vee \ \cos x \geq \dfrac{\sqrt{3}}{2} \\ \sin x \leq 0 \end{cases}$



$\pi + 2k\pi \leq x \leq \dfrac{7}{6}\pi + 2k\pi \quad \vee \quad \dfrac{11}{6}\pi + 2k\pi \leq x \leq 2\pi + 2k\pi$