1. $\displaystyle\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx$

$$t=\sqrt{x} \qquad dt=\frac{1}{2\sqrt{x}}dx \qquad =2\int e^t\,dt=2e^t+c=\mathbf{2e^{\sqrt{x}}+c}$$

2. $\displaystyle\int\left(\frac{4}{x}+\tan^2 x\right)dx$

$$=4\int\frac{1}{x}\,dx+\int(\tan^2 x+1-1)\,dx=\mathbf{4\ln|x|+\tan x-x+c}$$

3. $\displaystyle\int\left(\frac{1}{3\sqrt[3]{x}}-\frac{7}{6\sqrt[6]{x^5}}\right)dx$

$$=\frac{1}{3}\int x^{-\frac{1}{3}}\,dx-\frac{7}{6}\int x^{-\frac{5}{6}}\,dx=\frac{1}{3}\cdot\frac{1}{-\frac{1}{3}+1}\,x^{\frac{2}{3}}-\frac{7}{6}\cdot\frac{1}{1-\frac{5}{6}}\,x^{\frac{1}{6}}+c=\mathbf{\frac{\sqrt[3]{x^2}}{2}-7\sqrt[6]{x}+c}$$

4. $\displaystyle\int\frac{18x^3-26x}{3x-4}\,dx$

$18x^3$		$-26x$		$3x-4$
$-18x^3$	$24x^2$			$6x^2+8x+2$
	$24x^2$	$-26x$		
	$-24x^2$	$32x$		
		$6x$		
		$-6x$	$+8$	
			$+8$	

$$=\int\left(6x^2+8x+2+\frac{8}{3x-4}\right)dx=$$

$$=6\int x^2\,dx+8\int x\,dx+\int 2\,dx+\frac{8}{3}\int\frac{3}{3x-4}\,dx=$$

$$=\mathbf{2x^3+4x^2+2x+\frac{8}{3}\ln|3x-4|+c}$$

5. $\displaystyle\int\left(\frac{5}{x^6}+\frac{x^4-3x^2}{5}\right)dx$

$$=5\int x^{-6}\,dx+\frac{1}{5}\int x^4\,dx-\frac{3}{5}\int x^2\,dx=\frac{5}{-5}x^{-5}+\frac{1}{5}\cdot\frac{1}{5}x^5-\frac{3}{5}\cdot\frac{1}{3}x^3+c=\mathbf{-\frac{1}{x^5}+\frac{x^5}{25}-\frac{x^3}{5}+c}$$

6. $\displaystyle\int\frac{2x^3-15x^2-8x}{x-8}\,dx$

$$=\int\frac{x\,(2x^2-15x-8)}{x-8}dx=\int\frac{x\,(2x+1)(x-8)}{x-8}\,dx=\int(2x^2+x)\,dx=\mathbf{\frac{2x^3}{3}+\frac{x^2}{2}+c}$$

7. $\int \frac{\tan^2 x + 1}{\tan x}\, dx$

$$= \ln\ |\tan x|\ + c$$

8. $\int \cot x \ln \sin x\ dx$

$$t = \ln \sin x \qquad dt = \cot x\, dx \qquad = \int t dt = \frac{1}{2}t^2 + c = \frac{1}{2}\ln^2 \sin x + c$$

9. $\int \frac{3x^2}{1+x^6}\, dx$

$$t = x^3 \qquad dt = 3x^2 dx \qquad \int \frac{1}{1+t^2}\, dt = arctg\ t + c = arctg\ x^3 + c$$

10. $\int e^x \cos x\ dx$

$$f(x) = \cos x \quad f'(x) = -\sin x \quad g'(x) = e^x \quad g(x) = e^x \qquad = e^x \cos x + \int e^x \sin x\ dx =$$

$$f(x) = \sin x \quad f'(x) = \cos x \quad g'(x) = e^x \quad g(x) = e^x \qquad = e^x \cos x + e^x \sin x - \int e^x \cos x\ dx$$

$$\int e^x \cos x\ dx = e^x \cos x + e^x \sin x - \int e^x \cos x\ dx \qquad \int e^x \cos x\ dx = \frac{e^x}{2}(\cos x + \sin x) + c$$

11. $\int \frac{2x+1}{4x^2 - 12x + 9}\, dx$

$$\int \frac{2x+1}{(2x-3)^2}\, dx = \int \frac{2x+1-4+4}{(2x-3)^2}\, dx = \frac{1}{2}\int 2\frac{2x-3}{(2x-3)^2}\, dx + 2\int \frac{2}{(2x-3)^2}\, dx = \frac{1}{2}\ln\ |2x-3| - \frac{2}{2x-3} + c$$

12. $\int \arctan(x-1)\ dx$

$$t = \arctan(x-1) \qquad x - 1 = \tan t \qquad x = \tan t + 1 \qquad dx = (\tan^2 t + 1)\, dt$$

$$\int t(\tan^2 t + 1)\, dt \qquad f(t) = t \quad f'(t) = 1 \quad g'(t) = \tan^2 t + 1 \quad g(t) = \tan t$$

$$= t \tan t + \int \frac{-\sin t}{\cos t}\, dt = t \tan t + \ln|\cos t| + c = (x-1)\arctan\,(x-1) - \frac{1}{2}\ln\,(x^2 - 2x + 2) + c$$

$$\tan t = \frac{\sin t}{\cos t} \qquad \tan^2 t = \frac{1-\cos^2 t}{\cos^2 t} \qquad \frac{1}{\cos^2 t} = \tan^2 t + 1 \qquad \cos t = \frac{1}{\sqrt{\tan^2 t + 1}} = \frac{1}{\sqrt{x^2 - 2x + 2}}$$

13. $\int \frac{2}{\sqrt{1-4x^2}}\, dx$

$$= \int \frac{2}{\sqrt{1-(2x)^2}}\, dx = \boldsymbol{arcsin\,(2x) + c}$$

14. $\int \frac{x(x-2)}{x^3-3x^2}\, dx$

$D\,(x^3-3x^2) = 3x^2-6x$

$$= \frac{1}{3}\int \frac{3x^2-6x}{x^3-3x^2}\, dx = \boldsymbol{\frac{1}{3}\ln|x^3-3x^2| + c}$$

15. $\int \frac{5}{16+25x^2}\, dx$

$$= 5\int \frac{1}{16\left(1+\left(\frac{5}{4}x\right)^2\right)}\, dx = \frac{1}{4}\int \frac{\frac{5}{4}}{1+\left(\frac{5}{4}x\right)^2}\, dx = \boldsymbol{\frac{1}{4}arctan\left(\frac{5}{4}x\right) + c}$$

16. $\int \frac{3x-7}{x^3-6x^2+12x-8}\, dx$

$$= \int \frac{3x-7}{(x-2)^2}\, dx =$$

$$\frac{3x-7}{(x-2)^2} = \frac{A}{x-2} + \frac{B}{(x-2)^2} + \frac{C}{(x-2)^3} = \frac{A\,(x-2)^2 + B\,(x-2) + C}{(x-2)^3}$$

$$\begin{cases} A = 0 \\ B - 4A = 3 \\ 4A - 2B + C = -7 \end{cases} \qquad \begin{cases} A = 0 \\ B = 3 \\ -6 + C = -7 \end{cases} \qquad \begin{cases} A = 0 \\ B = 3 \\ C = -1 \end{cases}$$

$$= \int \left(\frac{3}{(x-2)^2} - \frac{1}{(x-2)^3}\right) dx = \boldsymbol{-\frac{3}{x-2} + \frac{1}{2(x-2)^2} + c}$$

17. $\int \frac{e^{2x}}{e^{2x}-e^x}\, dx$

$$= \int \frac{e^{2x}}{e^x(e^x-1)} dx = \int \frac{e^x}{e^x-1} dx = \boldsymbol{ln\,|e^x-1| + c}$$