Determina il dominio dei seguenti radicali:

1. $\sqrt{x-2}+\sqrt[4]{x+2}+\sqrt[3]{x}$

$$\begin{cases} x-2\geq 0 \\ x+2\geq 0 \end{cases} \Longrightarrow \begin{cases} x\geq 2 \\ x\geq -2 \end{cases} \Longrightarrow \boldsymbol{x\geq 2}$$

2. $\sqrt{\frac{x-2}{x+4}}+\sqrt[7]{x-3}+\frac{1}{\sqrt[4]{x^2+4}}$

$$\frac{x-2}{x+4}\geq 0 \qquad \begin{matrix} N\geq 0: & x\geq 2 \\ D>0: & x>-4 \end{matrix}$$

	-4		2	
−		**−**		**+**
−		**+**		**+**
+		**−**		**+**

$$\boldsymbol{x\leq -4} \quad \vee \quad \boldsymbol{x>2}$$

3. $\sqrt{|x-3|-2}+\frac{2x+3}{\sqrt[3]{x-7}}$

$$\begin{cases} |x-3|-2\geq 0 \\ x-7\neq 0 \end{cases} \Rightarrow \begin{cases} x-3\leq -2 \quad \vee \quad x-3\geq 2 \\ x\neq 7 \end{cases}$$

$$\begin{cases} x\leq 1 \quad \vee \quad x\geq 5 \\ x\neq 7 \end{cases}$$

$$\boldsymbol{x\leq 1} \quad \vee \quad \boldsymbol{5\leq x<7} \quad \vee \quad \boldsymbol{x>7}$$

Semplifica le seguenti espressioni numeriche:

4. $\left(\sqrt{3}+\sqrt{2}\right)^2+\left(\sqrt{2}-\sqrt{3}\right)\left(\sqrt{2}+\sqrt{3}\right)-\sqrt{6}\cdot\sqrt{(-2)^2}$

$3+2+2\sqrt{6}+2-3-2\sqrt{6}=\boldsymbol{4}$

5. $\left(\sqrt{32}+\sqrt{72}\right):\sqrt{2}-\left(\sqrt{5}+1\right)^2+\frac{(\sqrt{5}-1)(\sqrt{5}+1)}{(2-\sqrt{3})(2+\sqrt{3})}$

$\sqrt{16}+\sqrt{36}-\left(5+1+2\sqrt{5}\right)+\dfrac{5-1}{4-3}=4+6-5-1-2\sqrt{5}+4=\boldsymbol{8-2\sqrt{5}}$

6. $\left(\sqrt{3}-2\right)^3-\left(\sqrt{3}+2\right)^3$

$\left(\sqrt{3}\right)^3-6\left(\sqrt{3}\right)^2+12\sqrt{3}-8-\left[\left(\sqrt{3}\right)^3+6\left(\sqrt{3}\right)^2+12\sqrt{3}+8\right]=$

$=\left(\sqrt{3}\right)^3-18+12\sqrt{3}-8-\left(\sqrt{3}\right)^3-18-12\sqrt{3}-8=\mathbf{-52}$

7. $\frac{\sqrt{15}+\sqrt{5}+\sqrt{3}+1}{1+\sqrt{3}}+\frac{\sqrt{10}+2\sqrt{2}-\sqrt{5}-2}{1-\sqrt{2}}+1$

$=\frac{\sqrt{5}\left(\sqrt{3}+1\right)+1\left(\sqrt{3}+1\right)}{\sqrt{3}+1}+\frac{\sqrt{2}\left(\sqrt{5}+2\right)-\left(\sqrt{5}+2\right)}{-\left(\sqrt{2}-1\right)}+1=$

$=\frac{\left(\sqrt{3}+1\right)\left(\sqrt{5}+1\right)}{\sqrt{3}+1}+\frac{\left(\sqrt{5}+2\right)\left(\sqrt{2}-1\right)}{-\left(\sqrt{2}-1\right)}+1=\sqrt{5}+1-\sqrt{5}-2+1=\mathbf{0}$

8. $\sqrt[4]{\left(\sqrt{3}+3\right)\left(3-\sqrt{3}\right)+\left(1+\sqrt{3}\right)^2-\left(\sqrt{2\sqrt{3}}\right)^2-\left(\sqrt[3]{-6}\right)^3}$

$\sqrt[4]{9-3+1+3+2\sqrt{3}-2\sqrt{3}+6}=\sqrt[4]{16}=\sqrt[4]{2^4}=\mathbf{2}$

9. $\left(\sqrt[10]{32}-\sqrt[6]{3^6}\right)\left(\sqrt{2}+3\right)+\sqrt[6]{(-7)^6}-\sqrt{\left(1-\sqrt{3}\right)^2}+\sqrt[6]{27}+\sqrt[5]{-1}$

$\left(\sqrt[10]{2^5}-3\right)\left(\sqrt{2}+3\right)+\sqrt[6]{7^6}-\sqrt{\left(\sqrt{3}-1\right)^2}+\sqrt[6]{3^3}-\sqrt[5]{1}=$

$=\left(\sqrt{2}-3\right)\left(\sqrt{2}+3\right)+7-\sqrt{3}+1+\sqrt{3}-1=2-9+7=\mathbf{0}$

10. $\left[14\sqrt{2}:\left(2\sqrt{2}\right)-\frac{5\sqrt[4]{4}}{\sqrt{2}}+\sqrt{5}\right]\left(2-\sqrt[4]{25}\right)+3\cdot\sqrt[8]{\left(\frac{4}{9}\right)^3\cdot\left(\frac{2}{3}\right)^2}$

$\left[\frac{14\sqrt{2}}{2\sqrt{2}}-\frac{5\sqrt[4]{2^2}}{\sqrt{2}}+\sqrt{5}\right]\left(2-\sqrt[4]{5^2}\right)+3\cdot\sqrt[8]{\left(\frac{2}{3}\right)^6\cdot\left(\frac{2}{3}\right)^2}=$

$=\left[7-\frac{5\sqrt{2}}{\sqrt{2}}+\sqrt{5}\right]\left(2-\sqrt{5}\right)+3\cdot\sqrt[8]{\left(\frac{2}{3}\right)^8}=$

$=\left[7-5+\sqrt{5}\right]\left(2-\sqrt{5}\right)+3\cdot\frac{2}{3}=\left(2+\sqrt{5}\right)\left(2-\sqrt{5}\right)+2=4-5+2=\mathbf{1}$

Semplifica le seguenti espressioni letterali:

11. $\sqrt{2-x} \cdot \sqrt[3]{x-2}$

$$C.E.:\ 2-x \geq 0 \quad \Longrightarrow \quad x \leq 2$$

$$\sqrt{2-x} \cdot \sqrt[3]{-(2-x)} = -\sqrt{2-x} \cdot \sqrt[3]{2-x} = -\sqrt[6]{(2-x)^3 \cdot (2-x)^2} = -\sqrt[6]{(2-x)^5}$$

12. $\sqrt[4]{x^2+1} \cdot \sqrt[3]{x^2-1} \cdot \sqrt[6]{\frac{x-1}{x^2+1}}$

$$C.E.:\ x-1 \geq 0 \quad \Longrightarrow \quad x \geq 1$$

L'argomento della radice di indice 3 è sicuramente positivo, considerate le condizioni di esistenza:

$$\sqrt[12]{(x^2+1)^3 \cdot (x^2-1)^4 \cdot \left(\frac{x-1}{x^2+1}\right)^2} = \sqrt[12]{(x^2+1)^3 \cdot (x-1)^4 \cdot (x+1)^4 \cdot \frac{(x-1)^2}{(x^2+1)^2}} = \sqrt[12]{(x^2+1) \cdot (x-1)^6 \cdot (x+1)^4}$$

13. $\sqrt{\left(\sqrt{-x}+3\right)^2} - \sqrt[4]{(x)^2} + \sqrt[9]{(-x)^9} - 3$

$$C.E.: -x \geq 0 \quad \Longrightarrow \quad x \leq 0$$

$$\sqrt{-x} + 3 - \sqrt[4]{(-x)^2} - \sqrt[9]{x^9} - 3 = \sqrt{-x} + 3 - \sqrt{-x} - x - 3 = -x$$

Stabilisci M.C.D. e m.c.m. dei seguenti gruppi di polinomi, dopo averli scomposti:

14. x^2-3 $\qquad x\sqrt{3}-3$

$$x^2-3 = (x-\sqrt{3})(x+\sqrt{3}) \qquad x\sqrt{3}-3 = \sqrt{3}\,(x-\sqrt{3})$$

$$\boldsymbol{M.C.D.} = \boldsymbol{x-\sqrt{3}} \qquad \boldsymbol{m.c.m.} = \boldsymbol{\sqrt{3}\,(x-\sqrt{3})(x+\sqrt{3})}$$

15. $x^2-2x\sqrt{2}+2$ $\qquad x^2-2$ $\qquad 2-x\sqrt{2}$

$$x^2-2x\sqrt{2}+2 = \left(x-\sqrt{2}\right)^2 \qquad x^2-2 = (x-\sqrt{2})(x+\sqrt{2}) \qquad 2-x\sqrt{2} = -\sqrt{2}(x-\sqrt{2})$$

$$\boldsymbol{M.C.D.} = \boldsymbol{x-\sqrt{2}} \qquad \boldsymbol{m.c.m.} = \boldsymbol{\sqrt{2}\,\left(x-\sqrt{2}\right)^2(x+\sqrt{2})}$$