

1. $\lim_{x \rightarrow +\infty} [\log_2(x^2 + 1) - 2 \log_2 x] = \lim_{x \rightarrow +\infty} [\log_2(x^2 + 1) - \log_2 x^2] = \lim_{x \rightarrow +\infty} \log_2 \frac{x^2 + 1}{x^2} = \log_2 1 = \mathbf{0}$
2. $\lim_{x \rightarrow +\infty} \left(2^{\frac{1}{x}} - 2^{\frac{x-1}{x+1}} \right) = 2^0 - 2^1 = 1 - 2 = \mathbf{-1}$
3. $\lim_{x \rightarrow 0^+} \left(\frac{1}{x} - \frac{2}{x^2 + 4x} \right) = \lim_{x \rightarrow 0^+} \frac{x + 4 - 2}{x(x + 4)} = \mathbf{+\infty}$
4. $\lim_{x \rightarrow 2} \frac{x^3 - 4x}{x^3 - x^2 - 2x} = \lim_{x \rightarrow 2} \frac{x(x^2 - 4)}{x(x^2 - x - 2)} = \lim_{x \rightarrow 2} \frac{(x-2)(x+2)}{(x-2)(x+1)} = \frac{\mathbf{4}}{\mathbf{3}}$
5. $\lim_{x \rightarrow +\infty} \left(\frac{x+4}{2x+1} \right)^x = \lim_{x \rightarrow +\infty} \left(\frac{x}{2x} \right)^x = \lim_{x \rightarrow +\infty} \left(\frac{1}{2} \right)^x = \mathbf{0}$
6. $\lim_{x \rightarrow 2^+} \ln 3^{-\frac{x}{x-2}} = \mathbf{-\infty}$
7. $\lim_{x \rightarrow -\frac{\pi}{4}} \frac{\sin x + \cos x}{\cos 2x} = \lim_{x \rightarrow -\frac{\pi}{4}} \frac{\sin x + \cos x}{\cos^2 x - \sin^2 x} = \lim_{x \rightarrow -\frac{\pi}{4}} \frac{\sin x + \cos x}{(\cos x + \sin x)(\cos x - \sin x)} = \lim_{x \rightarrow -\frac{\pi}{4}} \frac{1}{\cos x - \sin x} = \frac{1}{\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}} = \frac{\mathbf{\sqrt{2}}}{\mathbf{2}}$