

1. $\lim_{x \rightarrow +\infty} [\log_2(x^2 + 1) - \log_2 x] = \lim_{x \rightarrow +\infty} \log_2 \frac{x^2+1}{x} = +\infty$
2. $\lim_{x \rightarrow +\infty} \left(2^{\frac{x-1}{x+1}} - 2^{\frac{1}{x}}\right) = 2^1 - 2^0 = 2 - 1 = 1$
3. $\lim_{x \rightarrow 0^-} \left(\frac{1}{x} + \frac{2}{x^2+4x}\right) = -\infty$
4. $\lim_{x \rightarrow -2} \frac{x^3-4x}{x^3+3x^2+2x} = \lim_{x \rightarrow -2} \frac{x(x^2-4)}{x(x^2+3x+2)} = \lim_{x \rightarrow -2} \frac{(x-2)(x+2)}{(x+2)(x+1)} = 4$
5. $\lim_{x \rightarrow -\infty} \left(\frac{x+4}{2x+1}\right)^x = \lim_{x \rightarrow -\infty} \left(\frac{x}{2x}\right)^x = \lim_{x \rightarrow -\infty} \left(\frac{1}{2}\right)^x = +\infty$
6. $\lim_{x \rightarrow 2^-} \ln 3^{-\frac{x}{x-2}} = +\infty$
7. $\lim_{x \rightarrow \frac{\pi}{4}} \frac{\sin x - \cos x}{\cos 2x} = \lim_{x \rightarrow \frac{\pi}{4}} \frac{\sin x - \cos x}{\cos^2 x - \sin^2 x} = \lim_{x \rightarrow \frac{\pi}{4}} \frac{-(-\sin x + \cos x)}{(\cos x + \sin x)(\cos x - \sin x)} = \lim_{x \rightarrow \frac{\pi}{4}} \frac{-1}{\cos x + \sin x} = \frac{-1}{\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}} = -\frac{\sqrt{2}}{2}$