Risolvi e discuti le seguenti equazioni letterali:

1. $\dfrac{a-x-1}{2} + \dfrac{x-2}{a} = \dfrac{2-x}{a^2-a}$

$\dfrac{a-x-1}{2} + \dfrac{x-2}{a} - \dfrac{2-x}{a(a-1)} = 0$

$\dfrac{a(a-1)(a-x-1) + 2(a-1)(x-2) - 2(2-x)}{2a(a-1)} = 0$ $\qquad\qquad C.E.: a \neq 0 \;\wedge\; a \neq 1$

$a^3 - a^2x - a^2 - a^2 + ax + a + 2ax - 4a - 2x + 4 - 4 + 2x = 0$

$3ax - a^2x = -a^3 + 2a^2 + 3a \qquad\qquad -ax(a-3) = -a(a^2 - 2a - 3) \qquad\qquad ax(a-3) = a(a-3)(a+1)$

$Se\,(a = 0 \;\vee\; a = 1) \qquad l'equazione\ perde\ significato$

$Se\ a = 3 \qquad l'equazione\ \grave{e}\ INDETERMINATA$

$Se\,(a \neq 0 \;\wedge\; a \neq 1 \;\wedge\; a \neq 3) \qquad x = a + 1$

2. $(2b - a)\,x + a = b - (a + 3b)\,x$

$2bx - ax + a = b - ax - 3bx \qquad\qquad 5bx = b - a$

$Se\ a = b = 0 \qquad l'equazione\ \grave{e}\ INDETERMINATA$

$Se\,(b = 0 \;\wedge\; a \neq 0) \qquad l'equazione\ \grave{e}\ IMPOSSIBILE$

$Se\ b \neq 0 \qquad x = \dfrac{b-a}{5b}$

Risolvi le seguenti equazioni e disequazioni:

3. $\dfrac{1+x}{2x-2} = -\dfrac{1}{6} + \dfrac{1}{1-x}$

$\dfrac{1+x}{2(x-1)} + \dfrac{1}{6} - \dfrac{1}{-(x-1)} = 0 \qquad\qquad \dfrac{3(1+x) + 1(x-1) + 6}{6(x-1)} = 0 \qquad\qquad C.A.: x \neq 1$

$3 + 3x + x - 1 + 6 = 0 \qquad\quad 4x = -8 \qquad\quad \mathbf{\color{blue}{x = -2}} \qquad\quad acc.$

4. $\left[\left(\dfrac{x}{2} - \dfrac{2}{x}\right) \cdot \dfrac{2x}{x^2 - 4x + 4} + 1\right]^3 \left(\dfrac{x-2}{x} - \dfrac{x}{x-2} + \dfrac{2x}{2x-4}\right)^2 = \dfrac{16}{x-2}$

$\left[\dfrac{x^2-4}{2x} \cdot \dfrac{2x}{(x-2)^2} + 1\right]^3 \left(\dfrac{x-2}{x} - \dfrac{x}{x-2} + \dfrac{2x}{2(x-2)}\right)^2 = \dfrac{16}{x-2}$

$\left(\dfrac{(x-2)(x+2)}{2x} \cdot \dfrac{2x}{(x-2)^2} + 1\right)^3 \left(\dfrac{x-2}{x} - \dfrac{x}{x-2} + \dfrac{x}{x-2}\right)^2 - \dfrac{16}{x-2} = 0$

$\left(\dfrac{x+2}{x-2} + 1\right)^3 \cdot \dfrac{(x-2)^2}{x^2} - \dfrac{16}{x-2} = 0 \qquad\qquad \left(\dfrac{x+2+x-2}{x-2}\right)^3 \cdot \dfrac{(x-2)^2}{x^2} - \dfrac{16}{x-2} = 0$

$\dfrac{8x^3}{(x-2)^3} \cdot \dfrac{(x-2)^2}{x^2} - \dfrac{16}{x-2} = 0 \qquad\qquad \dfrac{8x}{x-2} - \dfrac{16}{x-2} = 0 \qquad\qquad C.A.: \begin{cases} x \neq 0 \\ x \neq 2 \end{cases}$

$8x - 16 = 0 \qquad\qquad x = 2 \qquad soluzione\ non\ accettabile\ per\ le\ C.A. \qquad \Rightarrow \qquad \mathbf{\color{blue}{\nexists x \in \mathbb{R}}}$

5. $\dfrac{1-\frac{2}{3}x}{4} - \dfrac{2-\frac{1}{3}x}{5} < 1 - \dfrac{\frac{1}{2}-3x}{10}$

   $5\left(1-\dfrac{2}{3}x\right) - 4\left(2-\dfrac{1}{3}x\right) < 20 - 2\left(\dfrac{1}{2}-3x\right)$ $\qquad$ $5 - \dfrac{10}{3}x - 8 + \dfrac{4}{3}x < 20 - 1 + 6x$

   $-2x - 6x < +3 + 19$ $\qquad$ $-8x < 22$ $\qquad$ $\boldsymbol{x > -\dfrac{11}{4}}$

6. $(2x+1)^2 - 2x\,(x-3) \geq 2x\,(x+6) - 2\,x$

   $4x^2 + 4x + 1 - 2x^2 + 6x \geq 2x^2 + 12x - 2x$

   $10x - 12x + 2x \geq -1$ $\qquad$ $0x \geq -1$ $\qquad$ $\boldsymbol{\forall x \in \mathbb{R}}$

7. $1 - x(x+2) - [x - 3(1-2x)] > (7-x)(7+x)$

   $1 - x^2 - 2x - (x - 3 + 6x) > 49 - x^2$ $\qquad$ $1 - x^2 - 2x - x + 3 - 6x > 49 - x^2$

   $-9x > 45$ $\qquad$ $\boldsymbol{x < -5}$

8. $x\left(4x + \dfrac{1}{3}\right) + 2 > 1 - 4x + (4x-1)(x-3)$

   $4x^2 + \dfrac{1}{3}x + 2 > 1 - 4x + 4x^2 - 12\,x - x + 3$ $\qquad$ $\dfrac{1}{3}x + 17x > 2$ $\qquad$ $\dfrac{52}{3}x > 2$ $\qquad$ $\boldsymbol{x > \dfrac{3}{26}}$