Semplifica le seguenti espressioni:

1. $$\frac{\sqrt[3]{2+\sqrt{3}}\cdot\sqrt[9]{\left(2-\sqrt{3}\right)^3}+\sqrt{5}-1}{\sqrt{5}-2}+\frac{1}{\sqrt{5}+2}+\frac{41}{2-3\sqrt{5}}$$

$$=\frac{\sqrt[3]{2+\sqrt{3}}\cdot\sqrt[3]{2-\sqrt{3}}+\sqrt{5}-1}{\sqrt{5}-2}+\frac{1}{\sqrt{5}+2}+\frac{41}{2-3\sqrt{5}}\cdot\frac{2+3\sqrt{5}}{2+3\sqrt{5}}=$$

$$=\frac{\sqrt[3]{4-3}+\sqrt{5}-1}{\sqrt{5}-2}+\frac{1}{\sqrt{5}+2}+\frac{41\left(2+3\sqrt{5}\right)}{4-45}=\frac{1+\sqrt{5}-1}{\sqrt{5}-2}+\frac{1}{\sqrt{5}+2}-\frac{41\left(2+3\sqrt{5}\right)}{41}=$$

$$=\frac{\sqrt{5}}{\sqrt{5}-2}+\frac{1}{\sqrt{5}+2}-\left(2+3\sqrt{5}\right)=\frac{5+2\sqrt{5}+\sqrt{5}-2}{5-4}-2-3\sqrt{5}=3+3\sqrt{5}-2-3\sqrt{5}=\mathbf{1}$$

2. $$\left(3\sqrt{8}+\sqrt{125}-2\sqrt{18}+4\sqrt{50}-\sqrt{45}\right):\left(\sqrt{5}+10\sqrt{2}\right)$$

$$=\left(6\sqrt{2}+5\sqrt{5}-6\sqrt{2}+20\sqrt{2}-3\sqrt{5}\right):\left(\sqrt{5}+10\sqrt{2}\right)=\frac{2\sqrt{5}+20\sqrt{2}}{\sqrt{5}+10\sqrt{2}}=\frac{2\left(\sqrt{5}+10\sqrt{2}\right)}{\sqrt{5}+10\sqrt{2}}=\mathbf{2}$$

3. $$\frac{\sqrt{6}+4\sqrt{3}}{\sqrt{10}+4\sqrt{5}}\left(\frac{2}{\sqrt{5}}+\frac{1}{\sqrt{5}}\right):\frac{\sqrt{27}}{15}$$

$$=\frac{\sqrt{3}(\sqrt{2}+4)}{\sqrt{5}(\sqrt{2}+4)}\cdot\frac{3}{\sqrt{5}}\cdot\frac{15}{3\sqrt{3}}=\mathbf{3}$$

4. $$\left[3\left(\sqrt[3]{2}+1\right)^2+\left(\sqrt[3]{2}-1\right)^3-3\left(1+\sqrt[6]{2}\right)\left(1-\sqrt[6]{2}\right)-1\right]\cdot\frac{\left(\sqrt{2}-1\right)\left(\sqrt{2}+1\right)}{\sqrt[3]{54}}$$

$$=\left[3\left(\sqrt[3]{4}+2\sqrt[3]{2}+1\right)+2-3\sqrt[3]{4}+3\sqrt[3]{2}-1-3\left(1-\sqrt[3]{2}\right)-1\right]\cdot\frac{2-1}{3\sqrt[3]{2}}=$$

$$=\left(3\sqrt[3]{4}+6\sqrt[3]{2}+3+2-3\sqrt[3]{4}+3\sqrt[3]{2}-1-3+3\sqrt[3]{2}-1\right)\cdot\frac{1}{3\sqrt[3]{2}}=\frac{12\sqrt[3]{2}}{3\sqrt[3]{2}}=\mathbf{4}$$

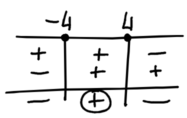
Determina il dominio delle seguenti funzioni:

5. $$y=\frac{\sqrt{16-x^2}}{\sqrt{2x-1}}$$

$$\begin{cases}16-x^2\geq 0\\ 2x-1>0\end{cases}$$

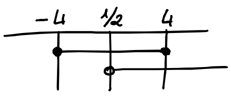
$(4-x)(4+x)\geq 0$

$IF\geq 0 \quad x\leq 4$
$IIF\geq 0 \quad x\geq -4$



$-4\leq x\leq 4$

$$\begin{cases}-4\leq x\leq 4\\ x>\dfrac{1}{2}\end{cases}$$



$\mathbf{\frac{1}{2}<x\leq 4}$

6. $y = \sqrt{\dfrac{4x - x^3}{x^3}} + \dfrac{\sqrt{3 - |x|}}{x + 1}$

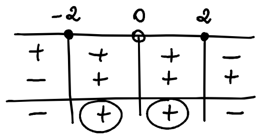
$$\begin{cases} \dfrac{4x - x^3}{x^3} \geq 0 \\ 3 - |x| \geq 0 \\ x + 1 \neq 0 \end{cases}$$

$\dfrac{x(4 - x^2)}{x^3} \geq 0 \qquad \dfrac{(2 - x)(2 + x)}{x^2} \geq 0$

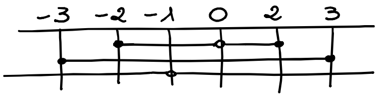
$N_1 \geq 0 \quad x \leq 2$

$N_2 \geq 0 \quad x \geq -2$

$D > 0 \quad x \neq 0$



$$\begin{cases} -2 \leq x < 0 \quad \vee \quad 0 < x \leq 2 \\ -3 \leq x \leq 3 \\ x \neq -1 \end{cases}$$



$\mathbf{-2 \leq x < -1 \quad \vee \quad -1 < x < 0 \quad \vee \quad 0 < x \leq 2}$

Risolvi i seguenti sistemi di equazioni:

7. $\begin{cases} x - 2\sqrt{2} = y\sqrt{2} \\ 2x + 2y\sqrt{2} = 8\sqrt{2} \end{cases}$

$+\begin{cases} x - y\sqrt{2} = 2\sqrt{2} \\ x + y\sqrt{2} = 4\sqrt{2} \end{cases}$ $\overline{2x = 6\sqrt{2}}$ $\Rightarrow \quad x = 3\sqrt{2}$

$-\begin{cases} x - y\sqrt{2} = 2\sqrt{2} \\ x + y\sqrt{2} = 4\sqrt{2} \end{cases}$ $\overline{-2y\sqrt{2} = -2\sqrt{2}}$ $\Rightarrow \quad y = 1$

$\begin{cases} \mathbf{x = 3\sqrt{2}} \\ \mathbf{y = 1} \end{cases}$

8. $\begin{cases} 2x\sqrt{3} + y = 3\sqrt{3} \\ \left(1 + \sqrt{3}\right)x = 2y + 1 - \sqrt{3} \end{cases}$

$+\begin{cases} 4x\sqrt{3} + 2y = 6\sqrt{3} \\ x + x\sqrt{3} - 2y = 1 - \sqrt{3} \end{cases}$ $\overline{5x\sqrt{3} + x = 5\sqrt{3} + 1}$ $\Rightarrow \quad x = \dfrac{5\sqrt{3} + 1}{5\sqrt{3} + 1} = 1$

$\begin{cases} x = 1 \\ 2\sqrt{3} + y = 3\sqrt{3} \end{cases}$

$\begin{cases} \mathbf{x = 1} \\ \mathbf{y = \sqrt{3}} \end{cases}$

Risolvi le seguenti equazioni e disequazioni:

9. $\dfrac{x - \sqrt{6}}{\sqrt{3} - 1} - \dfrac{x - \sqrt{2}}{\sqrt{3}} + \sqrt{2} = 0$

$\dfrac{\sqrt{3}(x - \sqrt{6}) - (\sqrt{3} - 1)(x - \sqrt{2}) + \sqrt{6}(\sqrt{3} - 1)}{\sqrt{3}\,(\sqrt{3} - 1)} = 0 \qquad x\sqrt{3} - 3\sqrt{2} - x\sqrt{3} + \sqrt{6} + x - \sqrt{2} + 3\sqrt{2} - \sqrt{6} = 0 \qquad \mathbf{x = \sqrt{2}}$

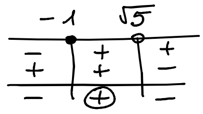
10. $\dfrac{1 - \sqrt{5}}{\sqrt{5} - x} = \dfrac{x^2 + \sqrt{5}}{x^2 - x\sqrt{5}} + \dfrac{1 - x}{x}$

$\dfrac{\sqrt{5} - 1}{x - \sqrt{5}} - \dfrac{x^2 + \sqrt{5}}{x(x - \sqrt{5})} - \dfrac{1 - x}{x} = 0 \qquad \dfrac{x\sqrt{5} - x - x^2 - \sqrt{5} - x + \sqrt{5} + x^2 - x\sqrt{5}}{x(x - \sqrt{5})} = 0 \qquad C.A.: \begin{cases} x \neq 0 \\ x \neq \sqrt{5} \end{cases}$

$x = 0 \qquad \textit{non accettabile per } C.A. \quad \Rightarrow \quad \nexists \mathbf{x} \in \mathbb{R}$

11. $\dfrac{1+\sqrt{5}}{\sqrt{5}-x} \geq 1$

$\dfrac{1+\sqrt{5}-\sqrt{5}+x}{\sqrt{5}-x} \geq 0$ $\qquad$ $\dfrac{1+x}{\sqrt{5}-x} \geq 0$ $\qquad$ $N \geq 0 \quad x \geq -1$; $\quad D > 0 \quad x < \sqrt{5}$



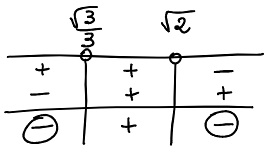
$\boldsymbol{-1 \leq x < \sqrt{5}}$

12. $\begin{cases} x\sqrt{3}-1 \leq 0 \\ \dfrac{\sqrt{2}-x}{3x-\sqrt{3}} < 0 \end{cases}$

$x\sqrt{3} \leq 1 \qquad x \leq \dfrac{1}{\sqrt{3}} \qquad x \leq \dfrac{\sqrt{3}}{3}$

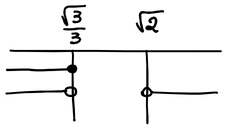
$N > 0 \quad x < \sqrt{2}$

$D > 0 \quad x > \dfrac{\sqrt{3}}{3}$



$x < \dfrac{\sqrt{3}}{3} \ \vee \ x > \sqrt{2}$

$\begin{cases} x \leq \dfrac{\sqrt{3}}{3} \\ x < \dfrac{\sqrt{3}}{3} \ \vee \ x > \sqrt{2} \end{cases}$



$\boldsymbol{x < \dfrac{\sqrt{3}}{3}}$