PRODOTTI NOTEVOLI

1. $$\frac{(a^{2x-y})^{2x+y}:(a^{2x-y})^{x+y}}{(a^{x-y})^{x-y}}=\frac{a^{4x^2-y^2}:a^{2x^2+xy-y^2}}{a^{x^2-2xy+y^2}}=\frac{a^{2x^2-xy}}{a^{x^2-2xy+y^2}}=a^{x^2+xy-y^2}$$

2. $$(1+2a^n)^2-(3a^n-1)^2-(-2a^n)^2+1=$$
$$=1+4a^n+4a^{2n}-9a^{2n}+6a^n-1-4a^{2n}+1=\ -9a^{2n}+10a^n+1$$

3. $$(x^n-y^n)^2-3x^n(y^n-x^n)-(2x^n+y^n)^2=$$
$$=x^{2n}-2x^ny^n+y^{2n}-3x^ny^n+3x^{2n}-4x^{2n}-4x^ny^n-y^{2n}=-9x^ny^n$$

4. $$[(x-3xy)(x+3xy)(x^2-1)(x^2+1)-9x^2y^2]:(-x^2)=$$
$$=[(x^2-9x^2y^2)(x^4-1)-9x^2y^2]:(-x^2)=(x^6-x^2-9x^6y^2+9x^2y^2-9x^2y^2):(-x^2)=$$
$$=-x^4+1+9x^4y^2$$

5. $$[(3a^2+5ab)(3a^2-5ab)-8a^3b]:(-3a^2)=$$
$$=(9a^4-25a^2b^2-8a^3b):(-3a^2)=-3a^2+\frac{25}{3}b^2+\frac{8}{3}ab$$

6. $$[(a^n-1)(a^n+1)]^2-(a^{2n}+2)^2+3(2a^{2n}+1)=$$
$$=(a^{2n}-1)^2-(a^{4n}+4a^{2n}+4)+6a^{2n}+3=$$
$$=a^{4n}-2a^{2n}+1-a^{4n}-4a^{2n}-4+6a^{2n}+3=0$$

7. $$(3-x^n)(1-x^n)-(2-x^n)^2+(1+x^n)(1-x^n)=$$
$$=3-3x^n-x^n+x^{2n}-4+4x^n-x^{2n}+1-x^{2n}=-x^{2n}$$

8. $$\left\{\left[\frac{(a^{n-2})^{2n-3}:(a^{-n})^{-n+3}:(a^n)^n}{a^{1-4n}}-\frac{2}{3}a^5-0{,}5\,a^{n-1}\right]^2-\frac{1}{9}(a^5)^2\right\}:(-2^2a^n)=$$
$$=\left\{\left[\frac{a^{2n^2-7n+6}:a^{-n^2-3n}:a^{n^2}}{a^{1-4n}}-\frac{2}{3}a^5-\frac{1}{2}a^{n-1}\right]^2-\frac{1}{9}a^{10}\right\}:(-4a^n)=$$
$$=\left\{\left[\frac{a^{-4n+6}}{a^{1-4n}}-\frac{2}{3}a^5-\frac{1}{2}a^{n-1}\right]^2-\frac{1}{9}a^{10}\right\}:(-4a^n)=$$
$$=\left\{\left[a^5-\frac{2}{3}a^5-\frac{1}{2}a^{n-1}\right]^2-\frac{1}{9}a^{10}\right\}:(-4a^n)=$$
$$=\left\{\left[\frac{1}{3}a^5-\frac{1}{2}a^{n-1}\right]^2-\frac{1}{9}a^{10}\right\}:(-4a^n)=$$
$$=\left\{\frac{1}{9}a^{10}-\frac{1}{3}a^{n+4}+\frac{1}{4}a^{2n-2}-\frac{1}{9}a^{10}\right\}:(-4a^n)=$$
$$=\left\{-\frac{1}{3}a^{n+4}+\frac{1}{4}a^{2n-2}\right\}:(-4a^n)=\frac{1}{12}a^4-\frac{1}{16}a^{n-2}$$

9. $$\left[\frac{(a^n)^{2n-1} : (a^{n-1})^{n+1} \cdot (a^{n+1})^{n+1}}{(a^{n+2})^{n-1}} + a^{n^2-4}\right]^2 - \frac{1}{2^{-1}} a^{2n^2} - a^{2n^2} \cdot \frac{a^{16}+1}{a^8} =$$

$$= \left[\frac{a^{2n^2-n} : a^{n^2-1} \cdot a^{n^2+2n+1}}{a^{n^2+n-2}} + a^{n^2-4}\right]^2 - 2a^{2n^2} - a^{2n^2} \cdot \left(a^8 + \frac{1}{a^8}\right) =$$

$$= \left[\frac{a^{2n^2+n+2}}{a^{n^2+n-2}} + a^{n^2-4}\right]^2 - 2a^{2n^2} - a^{2n^2+8} - a^{2n^2-8} =$$

$$= (a^{4+n^2} + a^{n^2-4})^2 - 2a^{2n^2} - a^{2n^2+8} - a^{2n^2-8} =$$

$$= a^{8+2n^2} + a^{2n^2-8} + 2a^{2n^2} - 2a^{2n^2} - a^{2n^2+8} - a^{2n^2-8} = 0$$

10. $$\left[\frac{1}{3} a^{2n-3} - \frac{a^{2n-1}}{a^{n-2}} - 2\,(a^{n-1})^3 : (a^2)^{n-2}\right]^3 =$$

$$= \left[\frac{1}{3} a^{2n-3} - a^{n+1} - 2a^{3n-3} : a^{2n-4}\right]^3 =$$

$$= \left[\frac{1}{3} a^{2n-3} - a^{n+1} - 2a^{n+1}\right]^3 =$$

$$= \left[\frac{1}{3} a^{2n-3} - 3a^{n+1}\right]^3 = \frac{1}{27} a^{6n-9} - a^{5n-5} + 9a^{4n-1} - 27a^{3n+3}$$

11. $$\left\{\left[\frac{(a^{n-2})^{n^2-1} : (a^n)^{n^2-1} \cdot (a^{2n})^n}{a^{2-n}} - 2\right]^4 - 16 - a^{3n}(a^n - 8)\right\} : (8a^n) =$$

$$= \left\{\left[\frac{a^{n^3-2n^2-n+2} : a^{n^3-n} \cdot a^{2n^2}}{a^{2-n}} - 2\right]^4 - 16 - a^{4n} + 8a^{3n}\right\} : (8a^n) =$$

$$= \left\{\left[\frac{a^2}{a^{2-n}} - 2\right]^4 - 16 - a^{4n} + 8a^{3n}\right\} : (8a^n) =$$

$$= \left\{\left[a^n - 2\right]^4 - 16 - a^{4n} + 8a^{3n}\right\} : (8a^n) =$$

$$= (a^{4n} - 8a^{3n} + 24a^{2n} - 32a^n + 16 - 16 - a^{4n} + 8a^{3n}) : (8a^n) =$$

$$= (+24a^{2n} - 32a^n) : (8a^n) = 3a^n - 4$$

12. $\left\{ a^n + a^{7n} + \left[0{,}5a^{2n-1} : (-2a^{n-2}) - \frac{3}{4} a^{n+1} \right]^7 : (-0{,}\bar{1}\, a^7) : 9 \right\}^4 =$

$= \left\{ a^n + a^{7n} + \left[\frac{1}{2} a^{2n-1} : (-2a^{n-2}) - \frac{3}{4} a^{n+1} \right]^7 : \left(-\frac{1}{9} a^7 \right) : 9 \right\}^4 =$

$= \left\{ a^n + a^{7n} + \left[-\frac{1}{4} a^{n+1} - \frac{3}{4} a^{n+1} \right]^7 : \left(-\frac{1}{9} a^7 \right) : 9 \right\}^4 =$

$= \left\{ a^n + a^{7n} - a^{7n+7} : \left(-\frac{1}{9} a^7 \right) : 9 \right\}^4 =$

$= (a^n + a^{7n} + a^{7n})^4 = (a^n + 2a^{7n})^4 = a^{4n} + 8a^{10n} + 24a^{16n} + 32a^{22n} + 16a^{28n}$

13. $\{[(2^{80} - 2^{79})^2 + 2^{157}] : 2^{150} - 2^7\} : 2^8 =$

$= \{[2^{160} - 2^{160} + 2^{158} + 2^{157}] : 2^{150} - 2^7\} : 2^8 = \{+2^8 + 2^7 - 2^7\} : 2^8 = \{+2^8\} : 2^8 = 1$

14. $(2^6 + 2^5 + 2^4)^2 - (2^6 - 2^5)(2^5 + 2^6) - 2^8 =$

$= 2^{12} + 2^{10} + 2^8 + 2^{12} + 2^{11} + 2^{10} - 2^{12} + 2^{10} - 2^8 =$

$= 2^{12} + 2^{10} + 2^{11} + 2^{10} + 2^{10} = 4 \cdot 2^{10} + 2^{10} + 2 \cdot 2^{10} + 2^{10} + 2^{10} = 9 \cdot 2^{10}$

15. $\dfrac{(3^{10} + 3^{11})^2}{(3^6 - 3^5)^3} =$

$= \dfrac{3^{20} + 3^{22} + 2 \cdot 3^{21}}{3^{18} - 3^{18} + 3^{17} - 3^{15}} = \dfrac{3^{20} + 9 \cdot 3^{20} + 6 \cdot 3^{20}}{9 \cdot 3^{15} - 3^{15}} = \dfrac{16 \cdot 3^{20}}{8 \cdot 3^{15}} = 2 \cdot 3^5$

16. $\dfrac{(2^6 + 2^7)^2}{(2^5 + 2^6 - 2^4)^2} =$

$= \dfrac{[2^6 (1+2)]^2}{[2^4 (2 + 2^2 - 1)]^2} = \dfrac{2^{12} \cdot 3^2}{2^8 \cdot 5^2} = \dfrac{2^4 \cdot 3^2}{5^2}$

17. $(a+b)^3 + (2a+b)^3 =$

$= a^3 + b^3 + 3a^2 b + 3ab^2 + 8a^3 + b^3 + 12a^2 b + 6ab^2 = 9a^3 + 2b^3 + 15a^2 b + 9ab^2$

18. $(3a + 2b)[a(2a+b) + (a+b)^2] =$

$= (3a+2b)[2a^2 + ab + a^2 + b^2 + 2ab] = (3a+2b)[3a^2 + 3ab + b^2] =$

$= 9a^3 + 9a^2 b + 3ab^2 + 6a^2 b + 6ab^2 + 2b^3 = 9a^3 + 2b^3 + 15a^2 b + 9ab^2$

19. $\left(\frac{3}{2}x^2-1\right)^3-3\left(\frac{3}{2}x^2-1\right)^2+\left(\frac{3}{2}x^2-1\right)^2\left(7-\frac{3}{2}x^2+1\right)=$

$=\frac{27}{8}x^6-\frac{27}{4}x^4+\frac{9}{2}x^2-1-3\left(\frac{9}{4}x^4-3x^2+1\right)+\left(\frac{9}{4}x^4-3x^2+1\right)\left(8-\frac{3}{2}x^2\right)=$

$=\frac{27}{8}x^6-\frac{27}{4}x^4+\frac{9}{2}x^2-1-\frac{27}{4}x^4+9x^2-3+18x^4-24x^2+8-\frac{27}{8}x^6+\frac{9}{2}x^4-\frac{3}{2}x^2=$

$=-12x^2+4+9x^4$

oppure, molto più semplicemente, basta vedere $\left(\frac{3}{2}x^2-1\right)=A$:

$A^3-3A^2+A^2(7-A)=A^3-3A^2+7A^2-A^3=4A^2$

$4\left(\frac{3}{2}x^2-1\right)^2=4\left(\frac{9}{4}x^4-3x^2+1\right)=9x^4-12x^2+4$