Semplifica le seguenti espressioni:

1. $\cos\alpha\ (1-\tan\alpha)+\sin\alpha\ (1-\cot\alpha)$

$$=\cos\alpha-\cos\alpha\cdot\frac{\sin\alpha}{\cos\alpha}+\sin\alpha-\sin\alpha\cdot\frac{\cos\alpha}{\sin\alpha}=\cos\alpha-\sin\alpha+\sin\alpha-\cos\alpha=\mathbf{0}$$

2. $(\sin^2\alpha+\tan^2\alpha)\ \cdot\csc^2\alpha-\frac{\cot^2\alpha+\csc^2\alpha}{\cot^2\alpha}+1$

$$=\left(\sin^2\alpha+\frac{\sin^2\alpha}{\cos^2\alpha}\right)\cdot\frac{1}{\sin^2\alpha}-\left(\frac{\cos^2\alpha}{\sin^2\alpha}+\frac{1}{\sin^2\alpha}\right)\cdot\frac{\sin^2\alpha}{\cos^2\alpha}+1=1+\frac{1}{\cos^2\alpha}-1-\frac{1}{\cos^2\alpha}+1=\mathbf{1}$$

3. $\left(1+\frac{\sec(-\alpha)}{\csc(\pi-\alpha)}\right)^2\cdot\frac{\csc^2\alpha}{\sec^2\left(\frac{\pi}{2}-\alpha\right)+2\sec\alpha\csc\alpha+\csc^2\left(\frac{\pi}{2}-\alpha\right)}+1$

$$=\left(1+\frac{\sec\alpha}{\csc\alpha}\right)^2\cdot\frac{\csc^2\alpha}{\csc^2\alpha+2\sec\alpha\csc\alpha+\sec^2\alpha}+1=$$
$$=\left(\frac{\csc\alpha+\sec\alpha}{\csc\alpha}\right)^2\cdot\frac{\csc^2\alpha}{(\csc\alpha+\sec\alpha)^2}+1=1+1=\mathbf{2}$$

4. $\cos^4\alpha+\sin^4\alpha+2+2\left(\frac{\sin\alpha}{\sec\alpha}\right)^2$

$$=\cos^4\alpha+\sin^4\alpha+2+2\left(\sin\alpha:\frac{1}{\cos\alpha}\right)^2=\cos^4\alpha+\sin^4\alpha+2+2\sin^2\alpha\cos^2\alpha=(\cos^2\alpha+\sin^2\alpha)^2+2=1+2=\mathbf{3}$$

5. $2\cdot\left(\cos^2\frac{\pi}{4}-\sin\frac{7}{6}\pi\right)^3-\left(\sqrt{3}\tan\frac{\pi}{3}-\cos 2\pi\right)^4+9\cdot\left(2\cos\frac{\pi}{4}-3\sin\frac{\pi}{4}\right)^{-2}$

$$=2\cdot\left(\frac{1}{2}+\frac{1}{2}\right)^3-\left(\sqrt{3}\cdot\sqrt{3}-1\right)^4+9\cdot\left(2\cdot\frac{\sqrt{2}}{2}-3\cdot\frac{\sqrt{2}}{2}\right)^{-2}=2-(3-1)^4+9\cdot\left(-\frac{\sqrt{2}}{2}\right)^{-2}=2-16+18=\mathbf{4}$$

6. Calcola: $\cos\left(arc\sin\frac{5}{13}\right)$

$$arc\sin\frac{5}{13}=\alpha\qquad\Rightarrow\qquad\sin\alpha=\frac{5}{13}\qquad con\ 0<\alpha<\frac{\pi}{2}$$
$$\cos\alpha=\sqrt{1-\sin^2\alpha}=\sqrt{1-\left(\frac{5}{13}\right)^2}=\mathbf{\frac{12}{13}}$$

7. Sapendo che $\tan\alpha=\frac{3}{4}$ con $\pi<\alpha<\frac{3}{2}\pi$, calcola $\sin\alpha$ e $\cos\alpha$.

$$\frac{\sin\alpha}{\cos\alpha}=\tan\alpha\quad\Rightarrow\quad\frac{\sin^2\alpha}{\cos^2\alpha}=\tan^2\alpha\quad\Rightarrow\quad\sin^2\alpha=\frac{9}{16}\cos^2\alpha\quad\Rightarrow\quad 16-16\cos^2\alpha=9\cos^2\alpha$$
$$\cos^2\alpha=\frac{16}{25}\cos^2\alpha\quad\Rightarrow\quad\cos\alpha=-\mathbf{\frac{4}{5}}\qquad\tan\alpha=\frac{\sin\alpha}{\cos\alpha}\quad\Rightarrow\quad\sin\alpha=\tan\alpha\cdot\cos\alpha=\frac{3}{4}\cdot\left(-\frac{4}{5}\right)=-\mathbf{\frac{3}{5}}$$

Determina i valori del parametro k in modo tale che siano verificate le seguenti uguaglianze:

8. $-(2+k^2)\sin x - k = 2 \qquad con \ \ \frac{3}{2}\pi < x < 2\pi$

$$\sin x = -\frac{k+2}{2+k^2} \qquad -1 < -\frac{k+2}{2+k^2} < 0$$

$$\begin{cases} -\dfrac{k+2}{k^2+2} > -1 \\ -\dfrac{k+2}{k^2+2} < 0 \end{cases} \qquad \begin{cases} \dfrac{k+2}{k^2+2} < 1 \\ \dfrac{k+2}{k^2+2} > 0 \end{cases} \qquad \begin{cases} \dfrac{k+2-k^2-2}{k^2+2} < 0 \\ k+2 > 0 \end{cases}$$

Una volta fatto il minimo comune multiplo, posso semplificare il denominatore, che è sempre positivo, trattandosi di una somma di quadrati:

$$\begin{cases} k^2 - k > 0 \\ k > -2 \end{cases} \qquad \begin{cases} k(k-1) > 0 \\ k > -2 \end{cases} \qquad \begin{cases} k < 0 \ \vee \ k > 1 \\ k > -2 \end{cases} \qquad \boldsymbol{-2 < k < 0 \ \vee \ k > 1}$$

9. $\sec x = \frac{k}{k+1} \qquad con \ 0 < x < \frac{\pi}{2}$

$$\frac{k}{k+1} > 1 \qquad \frac{k-k-1}{k+1} > 0 \qquad -\frac{1}{k+1} > 0 \qquad \frac{1}{k+1} < 0 \qquad k+1 < 0 \qquad \boldsymbol{k < -1}$$

10. $arc\cot\frac{2k+1}{k} = \frac{3}{4}\pi$

$$\frac{2k+1}{k} = -1 \qquad \frac{2k+1+k}{k} = 0 \qquad 3k+1 = 0 \qquad \boldsymbol{k = -\frac{1}{3}}$$

11. Calcola $\cos\left(arc\sin\sqrt{1-x^2}\right)$.

$$arc\sin\sqrt{1-x^2} = \alpha \qquad \sin\alpha = \sqrt{1-x^2} \qquad con -1 \le x \le 1 \ \ e \ \ 0 \le \alpha \le \frac{\pi}{2} \ \ dato\ che\ il\ seno\ è\ positivo:$$

$$\cos\alpha = \sqrt{1-\sin^2\alpha} = \sqrt{1-\left(\sqrt{1-x^2}\right)^2} = \sqrt{1-1+x^2} = \boldsymbol{|x|}$$

12. Verifica la seguente identità:

$$\frac{\cos\alpha}{\cos\alpha-2} + \frac{\cos^2\alpha+5\cos\alpha}{\sin^2\alpha-7\cos\alpha-11} = -\frac{4\cos\alpha}{4-\cos^2\alpha}$$

$$\frac{\cos\alpha}{\cos\alpha-2} + \frac{\cos\alpha(\cos\alpha+5)}{1-\cos^2\alpha-7\cos\alpha-11} = \frac{4\cos\alpha}{\cos^2\alpha-4}$$

$$\frac{\cos\alpha}{\cos\alpha-2} + \frac{\cos\alpha\ (\cos\alpha+5)}{-(\cos^2\alpha+7\cos\alpha+10)} = \frac{4\cos\alpha}{(\cos\alpha-2)(\cos\alpha+2)}$$

$$\frac{\cos\alpha}{\cos\alpha-2} - \frac{\cos\alpha\ (\cos\alpha+5)}{(\cos\alpha+2)(\cos\alpha+5)} = \frac{4\cos\alpha}{(\cos\alpha-2)(\cos\alpha+2)}$$

$$\frac{\cos\alpha}{\cos\alpha-2} - \frac{\cos\alpha}{\cos\alpha+2} = \frac{4\cos\alpha}{(\cos\alpha-2)(\cos\alpha+2)}$$

$$\frac{1}{\cos\alpha-2} - \frac{1}{\cos\alpha+2} = \frac{4}{(\cos\alpha-2)(\cos\alpha+2)}$$

$$\frac{\cos\alpha+2-\cos\alpha+2}{(\cos\alpha-2)(\cos\alpha+2)} = \frac{4}{(\cos\alpha-2)(\cos\alpha+2)}$$

$$\boldsymbol{\frac{4}{(\cos\alpha-2)(\cos\alpha+2)} = \frac{4}{(\cos\alpha-2)(\cos\alpha+2)}}$$

Stabilisci se le seguenti affermazioni sono vere o false:

	V	F
$\sqrt{\cos^2 \frac{5}{4}\pi} = \cos \frac{5}{4}\pi$	☐	☑
$2 \sin \frac{\pi}{4} = \sin \frac{\pi}{2}$	☐	☑
$\sin \frac{\pi}{18} = \cos \frac{4}{9}\pi$	☑	☐
$\sqrt{12 \cdot \cos^2 \frac{\pi}{3}} = 3$	☐	☑
$\sin^2 \frac{\pi}{2} = \sin \frac{\pi}{2}$	☑	☐
$\sin \frac{\pi}{4} \cdot \cos \frac{\pi}{4} = \frac{1}{2} \cdot \sin \frac{\pi}{2}$	☑	☐
$\cos \frac{\pi}{3} = \cos \left(-\frac{\pi}{3}\right)$	☑	☐
$\sin \frac{\pi}{3} = \sin \left(-\frac{\pi}{3}\right)$	☐	☑
$\dfrac{\tan \frac{\pi}{4}}{\sqrt{1 + \tan^2 \frac{\pi}{4}}} = -\sin \frac{\pi}{4}$	☐	☑
$\tan \frac{13}{12}\pi = \cot \frac{5}{12}\pi$	☑	☐
$\cot \frac{13}{12}\pi = \tan \frac{11}{12}\pi$	☐	☑
$\sqrt{\tan^2 \frac{17}{16}\pi} = \tan \frac{17}{16}\pi$	☑	☐
$arc \sin \left(-\frac{1}{2}\right) = \frac{7}{6}\pi$	☐	☑
$arc \sin \left(\frac{\sqrt{2}}{2}\right)^2 = \frac{\pi}{4}$	☐	☑
$arc \sin 1 = arc \cos 0$	☑	☐