

$$1. \quad \lim_{x \rightarrow +\infty} \frac{1 - 4^x}{1 - 2^x} = \lim_{x \rightarrow +\infty} \frac{(1 - 2^x)(1 + 2^x)}{1 - 2^x} = \lim_{x \rightarrow +\infty} (1 + 2^x) = +\infty$$

$$2. \quad \lim_{x \rightarrow 4} \frac{x - 1}{\sqrt{x^2 + 9}} = \frac{3}{5}$$

$$3. \quad \lim_{x \rightarrow -\infty} \frac{2x - \sqrt{4x^2 - x + 3}}{x - \sqrt{6 - x}} = \lim_{x \rightarrow -\infty} \frac{2x + 2x}{x} = 4$$

$$4. \quad \lim_{x \rightarrow \infty} \frac{x^2 - 3}{x^3 + x - 1} = \lim_{x \rightarrow \infty} \frac{x^2}{x^3} = 0$$

$$5. \quad \lim_{x \rightarrow +\infty} (\sqrt{x^2 + 8x + 5} - x) = \lim_{x \rightarrow +\infty} \frac{x^2 + 8x + 5 - x^2}{\sqrt{x^2 + 8x + 5} + x} = \lim_{x \rightarrow +\infty} \frac{8x}{x + x} = 4$$

$$6. \quad \lim_{x \rightarrow -\infty} \cos e^x = 1$$

$$7. \quad \lim_{x \rightarrow -\infty} \frac{\sqrt{x^2 + 1}}{x} = \lim_{x \rightarrow -\infty} \frac{|x|}{x} = \lim_{x \rightarrow -\infty} \frac{-x}{x} = -1$$

$$8. \quad \lim_{x \rightarrow +\infty} (-3x^4 + 5x^3 - x^2 - x + 2) = \lim_{x \rightarrow +\infty} (-3x^4) = -\infty$$

$$9. \quad \lim_{x \rightarrow 2} \frac{\sqrt{2} - \sqrt{x}}{x^2 - 4} = \lim_{x \rightarrow 2} \frac{-(\sqrt{x} - \sqrt{2})}{(\sqrt{x} - \sqrt{2})(\sqrt{x} + \sqrt{2})(x + 2)} = -\frac{1}{8\sqrt{2}} = -\frac{\sqrt{2}}{16}$$

$$10. \quad \lim_{x \rightarrow 0} \frac{\sqrt{1 + x^2} - 1}{x} = \lim_{x \rightarrow 0} \frac{x^2}{x(\sqrt{1 + x^2} + 1)} = 0$$

$$11. \quad \lim_{x \rightarrow +\infty} \arctan e^x = \frac{\pi}{2}$$

$$12. \quad \lim_{x \rightarrow +\infty} \left[ x \left( \frac{1}{2 + x} + 3x \right) \right] = +\infty$$

13.  $\lim_{x \rightarrow 4} \frac{\sqrt{2x+1}-3}{\sqrt{x-2}-\sqrt{2}} = \lim_{x \rightarrow 4} \frac{2(x-4)(\sqrt{x-2}+\sqrt{2})}{(x-4)(\sqrt{2x+1}+3)} = \lim_{x \rightarrow 4} \frac{2(\sqrt{x-2}+\sqrt{2})}{\sqrt{2x+1}+3} = \frac{4\sqrt{2}}{6} = \frac{2\sqrt{2}}{3}$
14.  $\lim_{x \rightarrow -1} \frac{x+1}{x^2-2x-3} = \lim_{x \rightarrow -1} \frac{x+1}{(x+1)(x-3)} = \lim_{x \rightarrow -1} \frac{1}{x-3} = -\frac{1}{4}$
15.  $\lim_{x \rightarrow \infty} \frac{x^5+7x^3-2}{3-x^5} = \lim_{x \rightarrow \infty} \frac{x^5}{-x^5} = -1$
16.  $\lim_{x \rightarrow -1} \frac{\sqrt[3]{x}+1}{x+1} = \lim_{x \rightarrow -1} \frac{\sqrt[3]{x}+1}{(\sqrt[3]{x}+1)(\sqrt[3]{x^2}-\sqrt[3]{x}+1)} = \frac{1}{3}$
17.  $\lim_{x \rightarrow 0^+} \left( \frac{5}{x} + \ln^2 x \right) = +\infty$
18.  $\lim_{x \rightarrow 1} \frac{x^3-3x+2}{x^2-2x+1} = \lim_{x \rightarrow 1} \frac{(x-1)(x^2+x-2)}{(x-1)^2} = \lim_{x \rightarrow 1} \frac{(x-1)(x+2)}{x-1} = 3$
19.  $\lim_{x \rightarrow +\infty} \frac{\sqrt[4]{1+x}}{\sin \frac{1}{x}} = +\infty$
20.  $\lim_{x \rightarrow \infty} \frac{3x^2+2x+1}{x^2+x+2} = \lim_{x \rightarrow \infty} \frac{3x^2}{x^2} = 3$
21.  $\lim_{x \rightarrow -\frac{2}{3}} \frac{3x^2+8x+4}{3x^2-7x-6} = \lim_{x \rightarrow -\frac{2}{3}} \frac{3x^2+6x+2x+4}{3x^2-9x+2x-6} = \lim_{x \rightarrow -\frac{2}{3}} \frac{(3x+2)(x+2)}{(3x+2)(x-3)} = \lim_{x \rightarrow -\frac{2}{3}} \frac{x+2}{x-3} = -\frac{4}{11}$
22.  $\lim_{x \rightarrow \infty} \cos \frac{1+x}{2+x^2} = \lim_{x \rightarrow \infty} \cos \frac{x}{x^2} = 1$
23.  $\lim_{x \rightarrow +\infty} \left( \sqrt{4x^2+x+1} - \sqrt{4x^2+1} \right) = \lim_{x \rightarrow +\infty} \frac{4x^2+x+1-4x^2-1}{\sqrt{4x^2+x+1}+\sqrt{4x^2+1}} = \lim_{x \rightarrow +\infty} \frac{x}{2x+2x} = \frac{1}{4}$
24.  $\lim_{x \rightarrow 0} \frac{e^{2x}+3e^x-4}{e^x-1} = \lim_{x \rightarrow 0} \frac{(e^x-1)(e^x+4)}{e^x-1} = 5$