Calcola i seguenti limiti:

1. $$\lim_{x\to 8}\frac{\sqrt[3]{x}-2}{x-8}=\lim_{x\to 8}\frac{\sqrt[3]{x}-2}{(\sqrt[3]{x}-2)(\sqrt[3]{x^2}+2\sqrt[3]{x}+4)}=\lim_{x\to 8}\frac{1}{\sqrt[3]{x^2}+2\sqrt[3]{x}+4}=\mathbf{\frac{1}{12}}$$

2. $$\lim_{x\to+\infty}\left(\frac{x^2+1}{x}\right)^{\pi}(1-x)=-\infty$$

3. $$\lim_{x\to-\infty}\ln\left(\frac{2^x+3}{2^{3-x}}\right)=\lim_{x\to-\infty}\ln\left(2^{2x-3}+3\cdot 2^{x-3}\right)=-\infty$$

4. $$\lim_{x\to+\infty}\frac{\ln(3x-1)-\ln 3x}{3x-1}=\lim_{x\to+\infty}\left[\left(\ln\frac{3x-1}{3x}\right)\left(\frac{1}{3x-1}\right)\right]=\mathbf{0}$$

5. $$\lim_{x\to+\infty}\left(\frac{3x-2}{x+1}\right)^{\frac{x-1}{2x}}=\sqrt{\mathbf{3}}$$

6. $$\lim_{x\to+\infty}\left(\frac{4x^2-x}{x+1}\right)^{x^2}=+\infty$$

7. $$\lim_{x\to+\infty}\log\frac{1}{x+2}=-\infty$$

8. $$\lim_{x\to 0}\frac{sen\,x}{x^2+2x}=\lim_{x\to 0}\frac{sen\,x}{x\,(x+2)}=\lim_{x\to 0}\left(\frac{sen\,x}{x}\cdot\frac{1}{x+2}\right)=\mathbf{\frac{1}{2}}$$

9. $$\lim_{x\to 0}\frac{sen\,x-2x}{sen\,x+x}=\lim_{x\to 0}\frac{x\left(\frac{sen\,x}{x}-2\right)}{x\left(\frac{sen\,x}{x}+1\right)}=-\mathbf{\frac{1}{2}}$$

10. $$\lim_{x\to+\infty}\left(\frac{x}{1+x}\right)^{-x}=\lim_{x\to+\infty}\left(\frac{1+x}{x}\right)^{x}=\lim_{x\to+\infty}\left(1+\frac{1}{x}\right)^{x}=\boldsymbol{e}$$

11. $$\lim_{x\to+\infty}\left(\frac{x+4}{2x+1}\right)^{x}=\mathbf{0}$$

12. $$\lim_{x\to\frac{\pi}{2}}\log_2\frac{\cos x+2\,sen\,x}{ctg\,x+1}=\log_2 2=\mathbf{1}$$

13. $$\lim_{x\to 0}\frac{2\,sen\,(1-e^x)}{e^x-1}=2\lim_{x\to 0}\frac{sen\,(1-e^x)}{-(1-e^x)}=\mathbf{-2}$$

14. $$\lim_{x\to 0}\frac{\ln(1+x)^{e^x}}{x}=\lim_{x\to 0}e^x\frac{\ln(1+x)}{x}=\lim_{x\to 0}\ln(1+x)^{\frac{1}{x}}=\lim_{y\to\infty}\ln\left(1+\frac{1}{y}\right)^{y}=\mathbf{1}$$

15. $$\lim_{x\to-\frac{\pi}{4}}\frac{sen\,x+\cos x}{\cos 2x}=\lim_{x\to-\frac{\pi}{4}}\frac{sen\,x+\cos x}{\cos^2 x-sen^2x}=\lim_{x\to-\frac{\pi}{4}}\frac{sen\,x+\cos x}{(\cos x-sen\,x)(\cos x+sen\,x)}=\lim_{x\to-\frac{\pi}{4}}\frac{1}{\cos x-sen\,x}=\mathbf{\frac{\sqrt{2}}{2}}$$

16. $\lim_{x\to 2^+} \ln 3^{-\frac{x}{x-2}} = \lim_{x\to 2^+} \frac{-x}{x-2}\ln 3 = -\infty$

17. $\lim_{x\to +\infty} \left(2^{\frac{1}{x}} - 2^{\frac{x-1}{x+1}}\right) = 1 - 2 = \mathbf{-1}$

18. $\lim_{x\to 2} \frac{x^3-4x}{x^3-2x^2+x-2} = \lim_{x\to 2} \frac{x\,(x^2-4)}{x^2\,(x-2)+(x-2)} = \lim_{x\to 2} \frac{x\,(x-2)(x+2)}{(x-2)(x^2+1)} = \mathbf{\frac{8}{5}}$

19. $\lim_{x\to -\infty} \frac{(x+2)^2}{(3x-1)^2} = \left(\lim_{x\to -\infty} \frac{x+2}{3x-1}\right)^2 = \left(\frac{1}{3}\right)^2 = \mathbf{\frac{1}{9}}$

20. $\lim_{x\to 0} \frac{\ln(1-2x)}{x} = \lim_{x\to 0} \ln\,(1-2x)^{\frac{1}{x}} = \lim_{y\to +\infty} \ln\left(1+\frac{1}{y}\right)^{-2y} = \mathbf{-2}$ $\qquad -2x = \frac{1}{y} \;\Rightarrow\; \frac{1}{x} = -2\,y$

21. $\lim_{x\to +\infty} \left(\frac{x+10}{x}\right)^{\frac{x}{2}} = \lim_{x\to +\infty} \left(1+\frac{10}{x}\right)^{\frac{x}{2}} = \lim_{y\to +\infty} \left(1+\frac{1}{y}\right)^{5y} = \mathbf{e^5}$ $\qquad \frac{10}{x} = \frac{1}{y} \;\Rightarrow\; x = 10\,y$

22. $\lim_{x\to 0} \frac{\ln(1+x)^2}{sen\,x} = 2\lim_{x\to 0} \left(\frac{\ln\,(1+x)}{x}\cdot\frac{x}{sen\,x}\right) = 2\lim_{x\to 0} \ln\,(1+x)^{\frac{1}{x}} = 2\lim_{y\to\infty} \ln\left(1+\frac{1}{y}\right)^{y} = \mathbf{2}$

23. $\lim_{x\to +\infty} \frac{2}{2-2^{\frac{x-1}{x}}} = +\infty$

24. $\lim_{x\to 1} \frac{4\,sen\ln x}{2^x\ln x} = 2\lim_{x\to 1} \frac{sen\ln x}{\ln x} = 2\lim_{\ln x\to 0} \frac{sen\ln x}{\ln x} = \mathbf{2}$