

Semplifica le seguenti espressioni:

$$\begin{aligned}
 1. \quad & \frac{\sqrt[3]{2+\sqrt{3}} \cdot \sqrt[9]{(2-\sqrt{3})^3} + \sqrt{5} - 1}{\sqrt{5} + 2} - \frac{1}{\sqrt{5} - 2} + \frac{29}{4 + 3\sqrt{5}} \\
 &= \frac{\sqrt[3]{2+\sqrt{3}} \cdot \sqrt[3]{2-\sqrt{3}} + \sqrt{5} - 1}{\sqrt{5} + 2} - \frac{1}{\sqrt{5} - 2} + \frac{29}{4 + 3\sqrt{5}} \cdot \frac{4 - 3\sqrt{5}}{4 - 3\sqrt{5}} = \\
 &= \frac{\sqrt[3]{4-3} + \sqrt{5} - 1}{\sqrt{5} + 2} - \frac{1}{\sqrt{5} - 2} + \frac{29(4 - 3\sqrt{5})}{16 - 45} = \frac{1 + \sqrt{5} - 1}{\sqrt{5} + 2} - \frac{1}{\sqrt{5} - 2} - \frac{29(4 - 3\sqrt{5})}{29} = \\
 &= \frac{\sqrt{5}}{\sqrt{5} + 2} - \frac{1}{\sqrt{5} - 2} - (4 - 3\sqrt{5}) = \frac{5 - 2\sqrt{5} - \sqrt{5} - 2}{5 - 4} - 4 + 3\sqrt{5} = 3 - 3\sqrt{5} - 4 + 3\sqrt{5} = -1
 \end{aligned}$$

$$\begin{aligned}
 2. \quad & (2\sqrt{18} - \sqrt{125} - 3\sqrt{8} + 4\sqrt{50} + \sqrt{45}) : (\sqrt{5} - 10\sqrt{2}) \\
 &= (6\sqrt{2} - 5\sqrt{5} - 6\sqrt{2} + 20\sqrt{2} + 3\sqrt{5}) : (\sqrt{5} - 10\sqrt{2}) = \frac{-2\sqrt{5} + 20\sqrt{2}}{\sqrt{5} - 10\sqrt{2}} = \frac{-2(\sqrt{5} - 10\sqrt{2})}{\sqrt{5} - 10\sqrt{2}} = -2
 \end{aligned}$$

$$\begin{aligned}
 3. \quad & \frac{\sqrt{6} + 5\sqrt{3}}{\sqrt{12} + 5\sqrt{6}} \left( -\frac{2}{\sqrt{6}} - \frac{1}{\sqrt{6}} \right) : \frac{\sqrt{27}}{18} \\
 &= \frac{\sqrt{3}(\sqrt{2} + 5)}{\sqrt{6}(\sqrt{2} + 5)} \cdot \left( -\frac{3}{\sqrt{6}} \right) \cdot \frac{18}{3\sqrt{3}} = -3
 \end{aligned}$$

$$\begin{aligned}
 4. \quad & [1 - 3(\sqrt[3]{2} + 1)^2 - (\sqrt[3]{2} - 1)^3 + 3(1 + \sqrt[6]{2})(1 - \sqrt[6]{2})] \cdot \frac{(\sqrt{2} - 1)(\sqrt{2} + 1)}{\sqrt[3]{54}} \\
 &= [1 - 3(\sqrt[3]{4} + 2\sqrt[3]{2} + 1) - (2 - 3\sqrt[3]{4} + 3\sqrt[3]{2} - 1) + 3(1 - \sqrt[3]{2})] \cdot \frac{2 - 1}{3\sqrt[3]{2}} = \\
 &= (1 - 3\sqrt[3]{4} - 6\sqrt[3]{2} - 3 - 2 + 3\sqrt[3]{4} - 3\sqrt[3]{2} + 1 + 3 - 3\sqrt[3]{2}) \cdot \frac{1}{3\sqrt[3]{2}} = \frac{-12\sqrt[3]{2}}{3\sqrt[3]{2}} = -4
 \end{aligned}$$

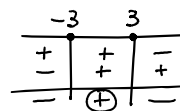
Determina il dominio delle seguenti funzioni:

$$5. \quad y = \frac{\sqrt{9 - x^2}}{\sqrt{2x - 1}}$$

$$\begin{cases} 9 - x^2 \geq 0 \\ 2x - 1 > 0 \end{cases}$$

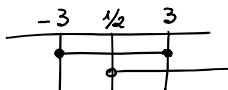
$$(3 - x)(3 + x) \geq 0$$

$$\begin{aligned}
 IF &\geq 0 & x &\leq 3 \\
 IIF &\geq 0 & x &\geq -3
 \end{aligned}$$



$$-3 \leq x \leq 3$$

$$\begin{cases} -3 \leq x \leq 3 \\ x > \frac{1}{2} \end{cases}$$



$$\frac{1}{2} < x \leq 3$$

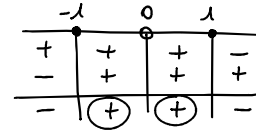
$$6. \quad y = \sqrt{\frac{x-x^3}{x^3}} + \frac{\sqrt{2-|x|}}{2x+1}$$

$$\begin{cases} \frac{x-x^3}{x^3} \geq 0 \\ 2-|x| \geq 0 \\ 2x+1 \neq 0 \end{cases}$$

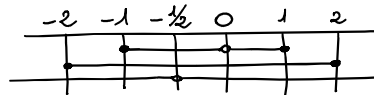
$$\frac{x(1-x^2)}{x^3} \geq 0$$

$$\frac{(1-x)(1+x)}{x^2} \geq 0$$

$$\begin{aligned} N_1 \geq 0 & \quad x \leq 1 \\ N_2 \geq 0 & \quad x \geq -1 \\ D > 0 & \quad x \neq 0 \end{aligned}$$



$$\begin{cases} -1 \leq x < 0 \vee 0 < x \leq 1 \\ -2 \leq x \leq 2 \\ x \neq -\frac{1}{2} \end{cases}$$



$$-1 \leq x < -\frac{1}{2} \vee -\frac{1}{2} < x < 0 \vee 0 < x \leq 1$$

Risolvi i seguenti sistemi di equazioni:

$$7. \quad \begin{cases} x\sqrt{2} - 2\sqrt{2} = y \\ 2x\sqrt{2} + 2y = 8\sqrt{2} \end{cases}$$

$$\begin{cases} x\sqrt{2} - y = 2\sqrt{2} \\ x\sqrt{2} + y = 4\sqrt{2} \end{cases} \Rightarrow x = 3$$

$$\begin{cases} x\sqrt{2} - y = 2\sqrt{2} \\ x\sqrt{2} + y = 4\sqrt{2} \end{cases} \Rightarrow y = \sqrt{2}$$

$$\begin{cases} x = 3 \\ y = \sqrt{2} \end{cases}$$

$$8. \quad \begin{cases} 2x\sqrt{3} + y = \sqrt{3} \\ (1 + \sqrt{3})x = 2y + 1 + 3\sqrt{3} \end{cases}$$

$$\begin{cases} 4x\sqrt{3} + 2y = 2\sqrt{3} \\ x + x\sqrt{3} - 2y = 1 + 3\sqrt{3} \end{cases} \Rightarrow x = \frac{5\sqrt{3} + 1}{5\sqrt{3} + 1} = 1$$

$$\begin{cases} x = 1 \\ 2\sqrt{3} + y = \sqrt{3} \end{cases}$$

$$\begin{cases} x = 1 \\ y = -\sqrt{3} \end{cases}$$

Risolvi le seguenti equazioni e disequazioni:

$$9. \quad \frac{x - \sqrt{6}}{\sqrt{2} - 1} - \frac{x - \sqrt{3}}{\sqrt{2}} + \sqrt{3} = 0$$

$$\frac{\sqrt{2}(x - \sqrt{6}) - (\sqrt{2} - 1)(x - \sqrt{3}) + \sqrt{6}(\sqrt{2} - 1)}{\sqrt{2}(\sqrt{2} - 1)} = 0 \quad x\sqrt{2} - 2\sqrt{3} - x\sqrt{2} + \sqrt{6} + x - \sqrt{3} + 2\sqrt{3} - \sqrt{6} = 0 \quad x = \sqrt{3}$$

$$10. \quad \frac{1 - \sqrt{7}}{\sqrt{7} - x} = \frac{x^2 + \sqrt{7}}{x^2 - x\sqrt{7}} + \frac{1 - x}{x}$$

$$\frac{\sqrt{7} - 1}{x - \sqrt{7}} - \frac{x^2 + \sqrt{7}}{x(x - \sqrt{7})} - \frac{1 - x}{x} = 0$$

$$\frac{x\sqrt{7} - x - x^2 - \sqrt{7} - x + \sqrt{7} + x^2 - x\sqrt{7}}{x(x - \sqrt{7})} = 0$$

$$C.A.: \begin{cases} x \neq 0 \\ x \neq \sqrt{7} \end{cases}$$

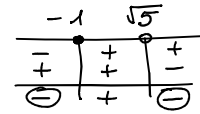
$$x = 0 \quad \text{non accettabile per C.A.} \Rightarrow \forall x \in \mathbb{R}$$

11.  $\frac{1 + \sqrt{5}}{\sqrt{5} - x} \leq 1$

$$\frac{1 + \sqrt{5} - \sqrt{5} + x}{\sqrt{5} - x} \leq 0$$

$$\frac{1 + x}{\sqrt{5} - x} \leq 0$$

$$\begin{aligned} N \geq 0 & \quad x \geq -1 \\ D > 0 & \quad x < \sqrt{5} \end{aligned}$$



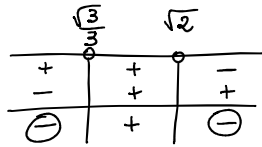
$$x \leq -1 \vee x > \sqrt{5}$$

12.  $\begin{cases} x\sqrt{3} - 1 \geq 0 \\ \frac{\sqrt{2} - x}{3x - \sqrt{3}} < 0 \end{cases}$

$$x\sqrt{3} \geq 1 \quad x \geq \frac{1}{\sqrt{3}}$$

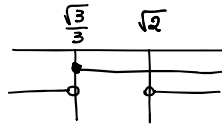
$$x \geq \frac{\sqrt{3}}{3}$$

$$\begin{aligned} N > 0 & \quad x < \sqrt{2} \\ D > 0 & \quad x > \frac{\sqrt{3}}{3} \end{aligned}$$



$$x < \frac{\sqrt{3}}{3} \vee x > \sqrt{2}$$

$$\begin{cases} x \geq \frac{\sqrt{3}}{3} \\ x < \frac{\sqrt{3}}{3} \vee x > \sqrt{2} \end{cases}$$



$$x > \sqrt{2}$$