1. $\left(\frac{6a}{a^2-9}+\frac{a}{a+3}+\frac{3}{3-a}\right)^3:\left(\frac{b}{b-2}+\frac{8}{4-b^2}-\frac{2}{b+2}\right)^4$

$=\left(\frac{6a}{(a-3)(a+3)}+\frac{a}{a+3}-\frac{3}{a-3}\right)^3:\left(-\frac{b}{2-b}+\frac{8}{(2-b)(2+b)}-\frac{2}{b+2}\right)^4=$ $\quad C.E.:\begin{cases}a\neq\pm3\\ b\neq\pm2\end{cases}$

$=\left(\frac{6a+a(a-3)-3(a+3)}{(a-3)(a+3)}\right)^3:\left(\frac{-b(2+b)+8-2(2-b)}{(2-b)(2+b)}\right)^4=$

$=\left(\frac{6a+a^2-3a-3a-9}{a^2-9}\right)^3:\left(\frac{-2b-b^2+8-4+2b}{4-b^2}\right)^4=\left(\frac{a^2-9}{a^2-9}\right)^3:\left(\frac{4-b^2}{4-b^2}\right)^4=1:1=\mathbf{1}$

2. $\frac{2y+2}{y^2-4}+\frac{3y}{y^2-4y+4}-\frac{4y^2+4y}{y^3-2y^2-4y+8}$

$=\frac{2y+2}{(y-2)(y+2)}+\frac{3y}{(y-2)^2}-\frac{4y^2+4y}{(y-2)(y^2-4)}=$

$=\frac{2y+2}{(y-2)(y+2)}+\frac{3y}{(y-2)^2}-\frac{4y^2+4y}{(y-2)^2(y+2)}=$ $\quad C.E.: y\neq\pm2$

$=\frac{(2y+2)(y-2)+3y(y+2)-(4y^2+4y)}{(y-2)^2(y+2)}=$

$=\frac{2y^2-4y+2y-4+3y^2+6y-4y^2-4y}{(y-2)^2(y+2)}=\frac{y^2-4}{(y-2)^2(y+2)}=\frac{(y-2)(y+2)}{(y-2)^2(y+2)}=\boldsymbol{\frac{1}{y-2}}$

3. $\left\{\left[\left(\frac{1}{a}+\frac{1}{b}-\frac{2}{a+b}\right):\left(\frac{a}{b}+\frac{b}{a}\right)\right]^2-\frac{2}{(a+b)^2}\right\}^3$

$=\left[\left(\frac{b(a+b)+a(a+b)-2ab}{ab(a+b)}:\frac{a^2+b^2}{ab}\right)^2-\frac{2}{(a+b)^2}\right]^3=$ $\quad C.E.:\begin{cases}a\neq-b\\ a\neq0\\ b\neq0\end{cases}$

$=\left[\left(\frac{ab+b^2+a^2+ab-2ab}{ab(a+b)}\cdot\frac{ab}{a^2+b^2}\right)^2-\frac{2}{(a+b)^2}\right]^3=$

$=\left[\left(\frac{a^2+b^2}{ab(a+b)}\cdot\frac{ab}{a^2+b^2}\right)^2-\frac{2}{(a+b)^2}\right]^3=\left(\frac{1}{(a+b)^2}-\frac{2}{(a+b)^2}\right)^3=\left(-\frac{1}{(a+b)^2}\right)^3=\boldsymbol{-\frac{1}{(a+b)^6}}$

4. $\left(\frac{x+5}{x^2+5x+6}+\frac{2}{x+3}-\frac{2}{x+2}\right)^2:\left(\frac{1}{x+2}\right)^3$

$=\left(\frac{x+5}{(x+2)(x+3)}+\frac{2}{x+3}-\frac{2}{x+2}\right)^2\cdot(x+2)^3=$ $\quad C.E.:\begin{cases}x\neq-2\\ x\neq-3\end{cases}$

$=\left(\frac{x+5+2(x+2)-2(x+3)}{(x+2)(x+3)}\right)^2\cdot(x+2)^3=\left(\frac{x+5+2x+4-2x-6}{(x+2)(x+3)}\right)^2\cdot(x+2)^3=$

$=\left(\frac{x+3}{(x+2)(x+3)}\right)^2\cdot(x+2)^3=\frac{1}{(x+2)^2}\cdot(x+2)^3=\boldsymbol{x+2}$

5. Eleva alla -2 il rapporto tra l'opposto di c e la differenza tra c e d e dividi il risultato per il reciproco dell'opposto del prodotto tra c e d.

$$\left(-\frac{c}{c-d}\right)^{-2}:\left(-\frac{1}{cd}\right)=\left(\frac{c-d}{-c}\right)^{2}\cdot(-cd)=\frac{(c-d)^2}{c^2}\cdot(-cd)=-\boldsymbol{\frac{d(c-d)^2}{c}} \qquad C.E.:\begin{cases} c\neq d \\ c\neq 0 \\ d\neq 0\end{cases}$$

6. Considera le seguenti frazioni algebriche:

$$A=\frac{x}{x-1} \qquad B=\frac{(x-1)^2}{x+1} \qquad C=\frac{(x+1)^2}{x^3}$$

Dopo aver stabilito per quali valori di $x\in\mathbb{R}$ esistono tutte e tre le frazioni, calcola ed esprimi in forma semplificata $D=A^2\cdot B\cdot C$.

$$A:\ C.E.:\ x\neq 1 \qquad B:\ C.E.:\ x\neq -1 \qquad C:\ C.E.:\ x\neq 0$$

$$D=\left(\frac{x}{x-1}\right)^2\cdot\frac{(x-1)^2}{x+1}\cdot\frac{(x+1)^2}{x^3}=\frac{x^2}{(x-1)^2}\cdot\frac{(x-1)^2}{x+1}\cdot\frac{(x+1)^2}{x^3}=\boldsymbol{\frac{x+1}{x}}$$

7. Considera il polinomio $P(x)=x^2-2x+3$. Calcola $\frac{P(a)-P(1)}{P(a+1)-P(0)}$ e trova le condizioni di esistenza.

$$\frac{P(a)-P(1)}{P(a+1)-P(0)}=\frac{a^2-2a+3-(1-2+3)}{(a+1)^2-2(a+1)+3-3}=\frac{a^2-2a+3-2}{a^2+2a+1-2a-2}=\frac{a^2-2a+1}{a^2-1}=\frac{(a-1)^2}{(a-1)(a+1)}=\boldsymbol{\frac{a-1}{a+1}}$$

$$C.E.: a\neq \pm 1$$