Calcola il valore delle seguenti espressioni:

1. $sen\frac{\pi}{2} + 2\,sen\pi - 3\,sen\frac{3}{2}\pi - 2\,sen\,0$

$$= 1 + 2\cdot 0 - 3\,(-1) - 2\cdot 0 = 1 + 3 = \mathbf{4}$$

2. $sen\,7\pi + \sqrt{2}sen\frac{\pi}{4} - sen\frac{3}{2}\pi + 4\,sen\frac{\pi}{6} - 5\,sen\,3\pi$

$$= 0 + \sqrt{2}\cdot\frac{\sqrt{2}}{2} - (-1) + 4\cdot\frac{1}{2} - 5\cdot 0 = 1 + 1 + 2 = \mathbf{4}$$

3. $8\cos\frac{\pi}{3} + 4\,sen\frac{\pi}{6} - \sqrt{2}\,sen\frac{\pi}{4} + \sqrt{2}\cos\frac{\pi}{4}$

$$= 8\cdot\frac{1}{2} + 4\cdot\frac{1}{2} - \sqrt{2}\cdot\frac{\sqrt{2}}{2} + \sqrt{2}\cdot\frac{\sqrt{2}}{2} = 4 + 2 - 1 + 1 = \mathbf{6}$$

4. $\dfrac{tg\,\pi - ctg\frac{3}{2}\pi - 3\,sen\left(-\frac{5}{2}\pi\right)}{sen^2\left(-\frac{\pi}{2}\right) + \cos^2(-\pi)}$

$$= \frac{0 - 0 - 3\cdot(-1)}{1 + 1} = \mathbf{\frac{3}{2}}$$

5. $\dfrac{a^2\left(sen\frac{\pi}{2} + \cos\frac{3}{2}\pi\right) - a\left[sen\left(-\frac{3}{2}\pi\right) + \cos(-2\pi)\right] - \left[sen\frac{3}{2}\pi - \cos\frac{\pi}{2}\right]}{a\,sen\left(-\frac{5}{2}\pi\right) + \cos(-4\pi)}$

$$= \frac{a^2(1+0) - a(1+1) - (-1-0)}{a\cdot(-1) + 1} = \frac{a^2 - 2a + 1}{1 - a} = \frac{(1-a)^2}{1-a} = \mathbf{1 - a} \quad \mathbf{con\ a \neq 1}$$

6. $5\,sen\frac{3}{2}\pi - 2\,tg\,\pi + \frac{4}{\sqrt{3}}\,tg\frac{\pi}{3} - 5\,ctg\frac{\pi}{4}$

$$= 5\cdot(-1) - 2\cdot 0 + \frac{4}{\sqrt{3}}\cdot\sqrt{3} - 5\cdot 1 = -5 + 4 - 5 = \mathbf{-6}$$

7. $tg\frac{\pi}{6}\left(sen\frac{\pi}{6} - \cos\frac{\pi}{3}\right) + ctg\frac{\pi}{3}\left(\cos 2\pi - sen\frac{\pi}{2}\right)$

$$= \frac{\sqrt{3}}{3}\left(\frac{1}{2} - \frac{1}{2}\right) + \frac{\sqrt{3}}{3}\,(1 - 1) = \mathbf{0}$$

8. $\dfrac{2\,tg\frac{\pi}{4} - tg\,2\pi + ctg\frac{\pi}{2}}{2\cos\frac{\pi}{6} - sen\frac{\pi}{2}}$

$$= \frac{2\cdot 1 - 0 + 0}{2\cdot\frac{\sqrt{3}}{2} - 1} = \frac{2}{\sqrt{3} - 1} = \frac{2\left(\sqrt{3} + 1\right)}{3 - 1} = \mathbf{\sqrt{3} + 1}$$

Verifica le seguenti identità nei loro domini, applicando le relazioni fondamentali:

9. $sen^2\alpha - 3\cos^2\alpha + 1 = 4\,sen^2\alpha - 2$

$$sen^2\alpha - 3\,(1 - sen^2\alpha) + 1 + 2 = 4\,sen^2\alpha$$

$$sen^2\alpha - 3 + 3\,sen^2\alpha + 3 = 4\,sen^2\alpha$$

$$\mathbf{4\,sen^2\alpha = 4\,sen^2\alpha}$$

10. $tg^2\alpha\cos^2\alpha + \frac{ctg^2\alpha}{sen^2\alpha} = \frac{sen^6\alpha - sen^2\alpha + 1}{sen^4\alpha}$

$$\frac{sen^2\alpha}{\cos^2\alpha}\cos^2\alpha + \frac{\cos^2\alpha}{sen^2\alpha}\cdot\frac{1}{sen^2\alpha} = sen^2\alpha + \frac{1 - sen^2\alpha}{sen^4\alpha}$$

$$\mathbf{sen^2\alpha + \frac{\cos^2\alpha}{sen^4\alpha} = sen^2\alpha + \frac{\cos^2\alpha}{sen^4\alpha}}$$

11. $(1 - sen\alpha)(1 + sen\,\alpha) - 2\,(sen^6\alpha + \cos^6\alpha) = 7\cos^2\alpha - 6\cos^4\alpha - 2$

$$1 - sen^2\alpha - 2\,sen^6\alpha - 2\cos^6\alpha = 7\cos^2\alpha - 6\cos^4\alpha - 2$$

$$\cos^2\alpha - 2\,(1 - \cos^2\alpha)^3 - 2\cos^6\alpha = 7\cos^2\alpha - 6\cos^4\alpha - 2$$

$$-2 + 6\cos^2\alpha - 6\cos^4\alpha + 2\cos^6\alpha - 2\cos^6\alpha = 6\cos^2\alpha - 6\cos^4\alpha - 2$$

$$\mathbf{0 = 0}$$

12. $\frac{ctg\,\alpha + 1}{sen\,\alpha - \cos\alpha} = -\frac{1 + tg\,\alpha}{sen\,\alpha\,(1 - tg\,\alpha)}$

$$(ctg\,\alpha + 1)(sen\,\alpha)(1 - tg\,\alpha) = -(1 + tg\,\alpha)(sen\alpha - cos\alpha)$$

$$sen\,\alpha\,(ctg\,\alpha - 1 + 1 - tg\,\alpha) = -sen\,\alpha + \cos\alpha - tg\,\alpha\,sen\,\alpha + tg\,\alpha\cos\alpha$$

$$sen\,\alpha\left(\frac{\cos\alpha}{sen\,\alpha} - \frac{sen\alpha}{\cos\alpha}\right) = -sen\,\alpha + \cos\alpha - \frac{sen\,\alpha}{\cos\alpha}\,sen\,\alpha + \frac{sen\,\alpha}{\cos\alpha}\cos\alpha$$

$$\cos\alpha - \frac{sen^2\alpha}{\cos\alpha} = -sen\,\alpha + \cos\alpha - \frac{sen^2\alpha}{\cos\alpha} + sen\,\alpha$$

$$\mathbf{\cos\alpha - \frac{sen^2\alpha}{\cos\alpha} = \cos\alpha - \frac{sen^2\alpha}{\cos\alpha}}$$

Determina le rimanenti funzioni goniometriche dell'arco α sapendo che:

13. $sen\ \alpha = \frac{1}{3} \qquad 0 < \alpha < \frac{\pi}{2}$

$$\cos\alpha = \sqrt{1 - sen^2\alpha} = \sqrt{1 - \frac{1}{9}} = \mathbf{\frac{2\sqrt{2}}{3}}$$

$$tg\ \alpha = \frac{sen\ \alpha}{\cos\alpha} = \frac{1}{3} : \frac{2\sqrt{2}}{3} = \frac{1}{2\sqrt{2}} = \mathbf{\frac{\sqrt{2}}{4}}$$

$$ctg\ \alpha = \frac{1}{tg\ \alpha} = \mathbf{2\sqrt{2}}$$

14. $tg\ \alpha = \frac{24}{7} \qquad \pi < \alpha < \frac{3}{2}\pi$

Ricavo la relazione che mi permette di calcolare seno e coseno:

$$tg\ \alpha = \frac{sen\ \alpha}{\cos\alpha} \quad \Rightarrow \quad tg\ \alpha\cos\alpha = sen\ \alpha$$

$$tg^2\alpha\cos^2\alpha = sen^2\alpha$$

$$tg^2\alpha\,(1 - sen^2\alpha) = sen^2\alpha$$

$$tg^2\alpha - tg^2\alpha\, sen^2\alpha = sen^2\alpha$$

$$sen^2\alpha\,(1 + tg^2\alpha) = tg^2\alpha$$

$$sen\ \alpha = \pm\frac{tg\ \alpha}{\sqrt{1 + tg^2\alpha}}$$

Perciò:

$$sen\ \alpha = -\frac{\frac{24}{7}}{\sqrt{1 + \frac{576}{49}}} = -\frac{24}{7}\cdot\frac{7}{25} = \mathbf{-\frac{24}{25}}$$

$$\cos\alpha = -\sqrt{1 - sen^2\alpha} = -\sqrt{1 - \frac{576}{625}} = \mathbf{-\frac{7}{25}}$$

$$ctg\ \alpha = \frac{1}{tg\ \alpha} = \mathbf{\frac{7}{24}}$$