Semplifica le seguenti espressioni supponendo che i valori delle variabili che in esse figurano soddisfino le condizioni di esistenza:

1. $[\sec\,(x+180°)\;ctg\,(-x-180°)\;sen\,(180°-x)-1]-[1-cosec\,(-x)\;tg\,(720°-x)\;(-\cos x)]$

$$=\left[\frac{1}{-\cos x}\,(-ctg\,x)\;senx-1\right]-\left[1-\frac{1}{-sen\,x}\,(-tg\,x)(-\cos x)\right]=$$

$$=\left[\frac{1}{-\cos x}\left(-\frac{\cos x}{sen\,x}\right)\;senx-1\right]-\left[1-\frac{1}{-sen\,x}\left(-\frac{sen\,x}{\cos x}\right)(-\cos x)\right]=(1-1)-(1+1)=\mathbf{-2}$$

2. $-2\,tg\,(\pi+\beta)\sec\,(9\pi+\beta)\;ctg\,(-\beta)+\frac{1}{2}\sec\,\frac{\pi}{4}\;ctg\,(7\pi+\beta)-2\,cosec\,(5\pi+\beta)\;tg\,(2\pi+\beta)$

$$=-2\,tg\,\beta\,\frac{1}{-cos\,\beta}\,(-ctg\,\beta)+\frac{1}{2}\cdot\frac{2}{\sqrt{2}}\,ctg\,\beta-2\,\frac{1}{-sen\,\beta}\,tg\,\beta=$$

$$=-2\,\frac{sen\,\beta}{\cos\beta}\cdot\frac{1}{-cos\,\beta}\left(-\frac{\cos\beta}{sen\,\beta}\right)+\frac{1}{\sqrt{2}}\,ctg\,\beta-2\,\frac{1}{-sen\,\beta}\cdot\frac{sen\,\beta}{\cos\beta}=$$

$$=-\frac{2}{\cos\beta}+\frac{\sqrt{2}}{2}ctg\,\beta+\frac{2}{\cos\beta}=\boldsymbol{\frac{\sqrt{2}}{2}ctg\,\beta}$$

3. $sen^2\left(\frac{\pi}{2}+\alpha\right)-\cos^2\,(3\pi-\alpha)+\cos\left(\frac{\pi}{2}-\alpha\right)\,sen\left(\frac{3}{2}\pi+\alpha\right)+4\,sen\left(\frac{3}{2}\pi-\alpha\right)[-sen\,(-\alpha)]$

$$=(\cos\,\alpha)^2-(-\cos\,\alpha)^2+sen\,\alpha\,(-\cos\,\alpha)+4\,(-\cos\,\alpha)\,sen\,\alpha=$$

$$=-sen\,\alpha\,\cos\,\alpha-4\,sen\,\alpha\,\cos\,\alpha=\boldsymbol{-5\,sen\,\alpha\,\cos\,\alpha}$$

Calcola il valore delle seguenti espressioni:

4. $\left(\sqrt{\frac{1-\cos\,300°}{1-\cos\,240°}}-\sqrt{\frac{1+sen\,150°}{2}}\right)^2$

$$=\left(\sqrt{\frac{1-\frac{1}{2}}{1+\frac{1}{2}}}-\sqrt{\frac{1+\frac{1}{2}}{2}}\right)^2=\left(\sqrt{\frac{1}{3}}-\sqrt{\frac{3}{4}}\right)^2=\left(\frac{\sqrt{3}}{3}-\frac{\sqrt{3}}{2}\right)^2=\left(\frac{\sqrt{3}}{6}\right)^2=\boldsymbol{\frac{1}{12}}$$

5. $\left[sen^2\frac{3}{4}\pi+\cos^2\left(-\frac{3}{4}\pi\right)\right]^4-\left[\sec^2\frac{8}{3}\pi-cosec^2\frac{11}{6}\pi\right]^3-\left[tg^2\frac{7}{4}\pi+ctg^2\frac{5}{6}\pi\right]^2$

$$=\left[\left(\frac{\sqrt{2}}{2}\right)^2+\left(-\frac{\sqrt{2}}{2}\right)^2\right]^4-\left[\left(\frac{1}{-\frac{1}{2}}\right)^2-\left(\frac{1}{-\frac{1}{2}}\right)^2\right]^3-\left[(-1)^2+\left(-\sqrt{3}\right)^2\right]^2=$$

$$=\left[\frac{1}{2}+\frac{1}{2}\right]^4-[(-2)^2-(-2)^2]^3-[1+3]^2=1-0-16=\mathbf{-15}$$

Verifica le seguenti identità supponendo che le variabili assumano valori per i quali le espressioni in esse contenute abbiano significato:

6. $sen\,(30°-x)+\cos\,(30°+x)=\frac{(1+\sqrt{3})\cos\,2x}{2\,(\cos\,x+sen\,x)}$

$$sen\,30°\cos\,x-\cos\,30°\,sen\,x+\cos\,30°\cos\,x-sen\,30°\,sen\,x=\frac{(1+\sqrt{3})(\cos^2\,x-sen^2\,x)}{2\,(\cos\,x+sen\,x)}$$

$$\frac{1}{2}\cos\,x-\frac{\sqrt{3}}{2}\,sen\,x+\frac{\sqrt{3}}{2}\cos\,x-\frac{1}{2}\,sen\,x=\frac{(1+\sqrt{3})(cos\,x-sen\,x)(cos\,x+sen\,x)}{2\,(\cos\,x+sen\,x)}$$

$$\frac{1+\sqrt{3}}{2}\cos\,x-\frac{1+\sqrt{3}}{2}\,sen\,x=\frac{(1+\sqrt{3})(cos\,x-sen\,x)}{2}$$

$$\mathbf{\frac{1+\sqrt{3}}{2}(cos\,x-sen\,x)=\frac{1+\sqrt{3}}{2}(cos\,x-sen\,x)}$$

7. $\frac{2\cos\,\alpha+sen\,2\alpha}{2\cos\,\alpha-sen\,2\alpha}=\frac{\sec\,\alpha+tg\,\alpha}{\sec\,\alpha-tg\,\alpha}$

$$\frac{2\cos\,\alpha\,+2\,sen\,\alpha\,\cos\,\alpha}{2\cos\,\alpha\,-2\,sen\,\alpha\,\cos\,\alpha}=\frac{\frac{1}{\cos\alpha}+\frac{sen\,\alpha}{\cos\alpha}}{\frac{1}{\cos\alpha}-\frac{sen\,\alpha}{\cos\alpha}}$$

$$\frac{2\cos\,\alpha\,(1+\,sen\,\alpha\,)}{2\cos\,\alpha\,(1-\,sen\,\alpha\,)}=\frac{\frac{1+\,sen\,\alpha}{\cos\alpha}}{\frac{1-\,sen\,\alpha}{\cos\alpha}}$$

$$\mathbf{\frac{1+\,sen\,\alpha}{1-\,sen\,\alpha}=\frac{1+\,sen\,\alpha}{1-\,sen\,\alpha}}$$

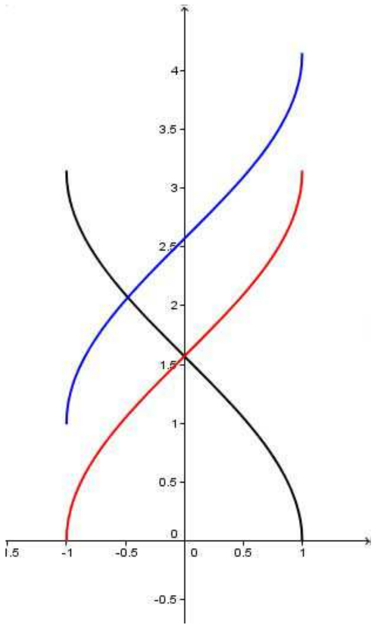
Traccia il grafico delle seguenti funzioni:

8. $y = \arccos(-x) + 1$ $\qquad y = \left|ctg\left(x - \frac{\pi}{6}\right) - 2\right|$ $\qquad y = cosec\frac{1}{2}x$

$y = \arccos x$

$y = \arccos(-x)$

$y = \arccos(-x) + 1$

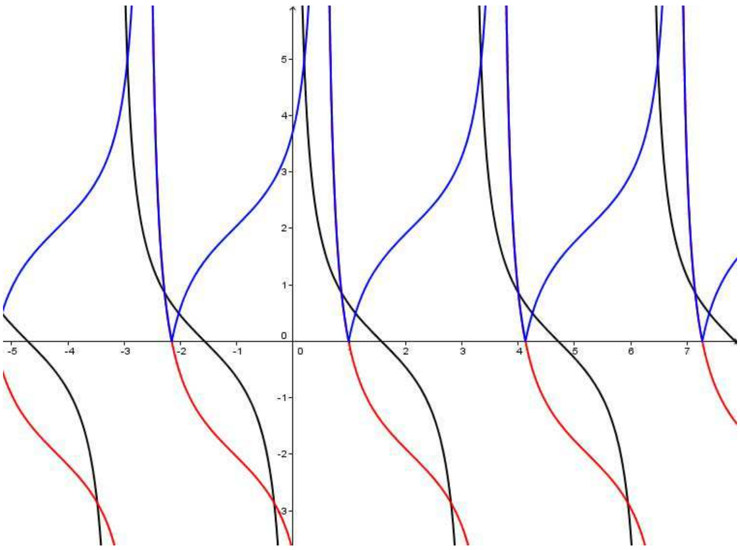


$y = ctg\, x$

Traslazione secondo il vettore $\vec{v}\left(\frac{\pi}{6}; -2\right)$

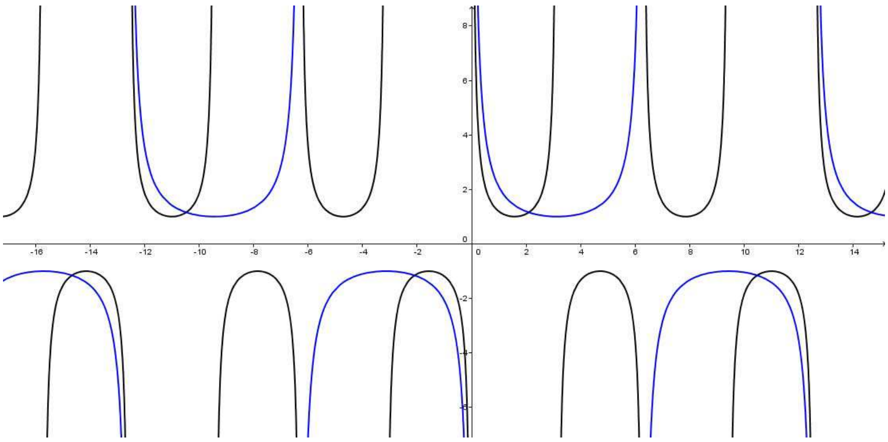
$y = ctg\left(x - \frac{\pi}{6}\right) - 2$

$y = \left|ctg\left(x - \frac{\pi}{6}\right) - 2\right|$



$y = cosec\, x$

$y = cosec\frac{1}{2}x$



Determina il dominio delle seguenti funzioni:

9. $y = arc\,\text{tg}\,(\log\,(x-1))$ $\quad y = ctg\left(x-\frac{\pi}{2}\right)$ $\quad y = arc\,\text{sen}\,(-3x+2)$

$y = arc\,\text{tg}\,(\log\,(x-1))$ $\quad x-1>0$ $\quad \boldsymbol{x>1}$

$y = ctg\left(x-\frac{\pi}{2}\right)$ $\quad x-\frac{\pi}{2}\neq 0+k\pi$ $\quad \boldsymbol{x\neq\frac{\pi}{2}+k\pi}$

$y = arc\,\text{sen}\,(-3x+2)$ $\quad -1\leq -3x+2\leq 1$ $\quad -3\leq -3x\leq -1$ $\quad \boldsymbol{\frac{1}{3}\leq x\leq 1}$