

Semplifica le seguenti espressioni:

$$\begin{aligned}
 1. \quad & (\sqrt{5} - \sqrt{2})^2 - (\sqrt{10} - 1)^2 + (\sqrt{6} + \sqrt{2})(\sqrt{6} - \sqrt{2}) - (\sqrt{2} + 1)(\sqrt{6} - \sqrt{3}) \\
 & = 5 + 2 - 2\sqrt{10} - (10 + 1 - 2\sqrt{10}) + 6 - 2 - \sqrt{3}(\sqrt{2} + 1)(\sqrt{2} - 1) = \\
 & = 5 + 2 - 2\sqrt{10} - 10 - 1 + 2\sqrt{10} + 6 - 2 - \sqrt{3} = -\sqrt{3}
 \end{aligned}$$

$$\begin{aligned}
 2. \quad & \left(1 + \frac{5 - \sqrt{2}}{5 + \sqrt{2}}\right) : \left(\frac{\sqrt{2} - 1}{2 + 5\sqrt{2}}\right) : (2 - \sqrt{2}) - \sqrt{900} - \sqrt{200} \\
 & = \frac{5 + \sqrt{2} + 5 - \sqrt{2}}{5 + \sqrt{2}} \cdot \frac{\sqrt{2}(\sqrt{2} + 5)}{\sqrt{2} - 1} \cdot \frac{1}{\sqrt{2}(\sqrt{2} - 1)} - 30 - 10\sqrt{2} = \\
 & = \frac{10}{(\sqrt{2} - 1)^2} \cdot \frac{(\sqrt{2} + 1)^2}{(\sqrt{2} + 1)^2} - 30 - 10\sqrt{2} = \\
 & = 10(2 + 1 + 2\sqrt{2}) - 30 - 10\sqrt{2} = 30 + 20\sqrt{2} - 30 - 10\sqrt{2} = 10\sqrt{2}
 \end{aligned}$$

$$\begin{aligned}
 3. \quad & \left(\frac{3 + \sqrt{6}}{\sqrt{2} + \sqrt{3}} + \frac{\sqrt{15} - \sqrt{3}}{\sqrt{5} - 1} - \sqrt{27} + 2\right)^2 (2 + \sqrt{3})^2 \\
 & = \left(\frac{\sqrt{3}(\sqrt{3} + \sqrt{2})}{\sqrt{2} + \sqrt{3}} + \frac{\sqrt{3}(\sqrt{5} - 1)}{\sqrt{5} - 1} - 3\sqrt{3} + 2\right)^2 (2 + \sqrt{3})^2 = \\
 & = (\sqrt{3} + \sqrt{3} - 3\sqrt{3} + 2)^2 (2 + \sqrt{3})^2 = (2 - \sqrt{3})^2 (2 + \sqrt{3})^2 = (4 - 3)^2 = 1
 \end{aligned}$$

$$\begin{aligned}
 4. \quad & \left[\frac{25(\sqrt{48} - \sqrt{12} + 5)}{2\sqrt{3}(\sqrt{7} - \sqrt{2}) + 5(\sqrt{7} - \sqrt{2})} - \sqrt{50}\right] : \sqrt{7} \\
 & = \left[\frac{25(4\sqrt{3} - 2\sqrt{3} + 5)}{(\sqrt{7} - \sqrt{2})(2\sqrt{3} + 5)} - 5\sqrt{2}\right] : \sqrt{7} = \left[\frac{25(2\sqrt{3} + 5)}{(\sqrt{7} - \sqrt{2})(2\sqrt{3} + 5)} \cdot \frac{\sqrt{7} + \sqrt{2}}{\sqrt{7} + \sqrt{2}} - 5\sqrt{2}\right] : \sqrt{7} = \\
 & = (5\sqrt{7} + 5\sqrt{2} - 5\sqrt{2}) : \sqrt{7} = 5\sqrt{7} : \sqrt{7} = 5
 \end{aligned}$$

Determina il dominio delle seguenti funzioni:

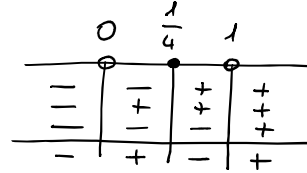
$$5. \quad y = \frac{\sqrt{x^2 + 4x + 4}}{\sqrt[3]{x^2 - 4}}$$

$$\begin{cases} x^2 + 4x + 4 \geq 0 \\ x^2 - 4 \neq 0 \end{cases} \quad \begin{cases} (x + 2)^2 \geq 0 \\ (x - 2)(x + 2) \neq 0 \end{cases} \quad \begin{cases} \forall x \in \mathbb{R} \\ x \neq \pm 2 \end{cases} \quad x \neq \pm 2$$

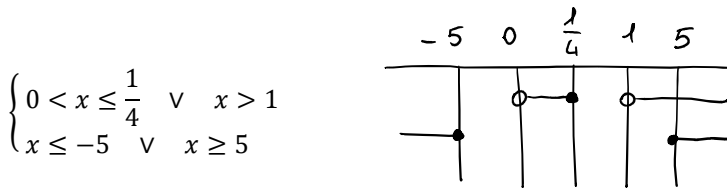
$$6. \quad y = \sqrt{\frac{1}{x} + \frac{3}{x-1}} + \sqrt{\frac{|x|-5}{x^2+7}}$$

$$\begin{cases} \frac{1}{x} + \frac{3}{x-1} \geq 0 \\ \frac{|x|-5}{x^2+7} \geq 0 \end{cases}$$

$$\frac{x-1+3x}{x(x-1)} \geq 0 \quad \frac{4x-1}{x(x-1)} \geq 0 \quad \begin{matrix} N \geq 0 & x \geq \frac{1}{4} \\ D_1 > 0 & x > 0 \\ D_2 > 0 & x > 1 \end{matrix}$$



$$\frac{|x|-5}{x^2+7} \geq 0 \quad \begin{matrix} N \geq 0 & |x|-5 \geq 0 \\ D > 0 & \forall x \in \mathbb{R} \end{matrix} \quad |x|-5 \geq 0 \quad \Rightarrow \quad x \leq -5 \vee x \geq 5$$



$$x \geq 5$$

Risolvi i seguenti sistemi di equazioni:

$$7. \quad \begin{cases} x\sqrt{3} + y\sqrt{7} = 27 \\ \frac{\sqrt{3}}{2}x + \frac{2\sqrt{7}}{3}y = 17 \end{cases}$$

$$\begin{cases} 3x\sqrt{3} + 3y\sqrt{7} = 81 \\ 3x\sqrt{3} + 4y\sqrt{7} = 102 \\ -y\sqrt{7} = -21 \end{cases} \Rightarrow y = \frac{21}{\sqrt{7}} \cdot \frac{\sqrt{7}}{\sqrt{7}} = 3\sqrt{7} \quad \begin{cases} x\sqrt{3} + 21 = 27 \\ y = 3\sqrt{7} \end{cases} \Rightarrow \begin{cases} x = \frac{6}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} \\ y = 3\sqrt{7} \end{cases} \quad \begin{cases} x = 2\sqrt{3} \\ y = 3\sqrt{7} \end{cases}$$

$$8. \quad \begin{cases} 2x\sqrt{2} - y\sqrt{3} = 6\sqrt{3} \\ 2x - y\sqrt{6} = 4\sqrt{6} \end{cases}$$

$$\begin{cases} 4x - y\sqrt{6} = 6\sqrt{6} \\ 2x - y\sqrt{6} = 4\sqrt{6} \\ -2x = 2\sqrt{6} \end{cases} \Rightarrow x = \sqrt{6} \quad \begin{cases} x = \sqrt{6} \\ 2\sqrt{6} - y\sqrt{6} = 4\sqrt{6} \end{cases} \quad \begin{cases} x = \sqrt{6} \\ y = -2 \end{cases}$$

Risolvi le seguenti equazioni e disequazioni:

$$9. \quad \frac{\sqrt{3}}{x+\sqrt{2}} - \frac{1}{x} = \frac{\sqrt{3}-1}{x-\sqrt{2}}$$

$$\frac{x\sqrt{3}(x-\sqrt{2}) - (x-\sqrt{2})(x+\sqrt{2}) - x(\sqrt{3}-1)(x+\sqrt{2})}{x(x+\sqrt{2})(x-\sqrt{2})} = 0 \quad C.A.: x \neq 0 \wedge x \neq \pm\sqrt{2}$$

$$x^2\sqrt{3} - x\sqrt{6} - x^2 + 2 - x^2\sqrt{3} - x\sqrt{6} + x^2 + x\sqrt{2} = 0 \quad -2x\sqrt{6} + x\sqrt{2} = -2$$

$$x = \frac{2}{2\sqrt{6}-\sqrt{2}} \cdot \frac{2\sqrt{6}+\sqrt{2}}{2\sqrt{6}+\sqrt{2}} \quad x = \frac{2\sqrt{6}+\sqrt{2}}{11}$$

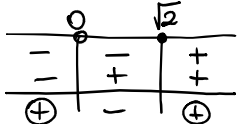
10.
$$\frac{x\sqrt{2}-1}{\sqrt{2}} = \frac{1-\sqrt{2}}{2-\sqrt{2}} + \frac{x-\sqrt{5}}{1-\sqrt{5}}$$

$$\frac{x\sqrt{2}-1}{\sqrt{2}} = \frac{-(\sqrt{2}-1)}{\sqrt{2}(\sqrt{2}-1)} - \frac{x-\sqrt{5}}{\sqrt{5}-1}$$
 $x - \frac{1}{\sqrt{2}} = -\frac{1}{\sqrt{2}} - \frac{x-\sqrt{5}}{\sqrt{5}-1}$ $x + \frac{x-\sqrt{5}}{\sqrt{5}-1} = 0$

$x(\sqrt{5}-1) + x - \sqrt{5} = 0$ $x\sqrt{5} - x + x - \sqrt{5} = 0$ $x = 1$

11.
$$\frac{1}{\sqrt{2}} \geq \frac{1}{x}$$

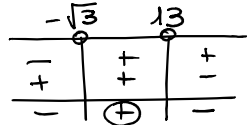
$$\frac{x-\sqrt{2}}{x\sqrt{2}} \geq 0$$
 $N \geq 0 \quad x \geq \sqrt{2}$
 $D > 0 \quad x > 0$



$x < 0 \vee x \geq \sqrt{2}$

12.
$$\begin{cases} \frac{x+\sqrt{3}}{13-x} > 0 \\ -\sqrt{3}x+6 \leq 0 \end{cases}$$

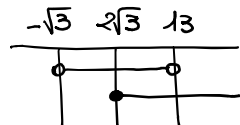
$N > 0 \quad x > -\sqrt{3}$
 $D > 0 \quad x < 13$



$-\sqrt{3} < x < 13$

$x\sqrt{3} \geq 6 \quad x \geq \frac{6}{\sqrt{3}} \quad x \geq 2\sqrt{3}$

$$\begin{cases} -\sqrt{3} < x < 13 \\ x \geq 2\sqrt{3} \end{cases}$$



$2\sqrt{3} \leq x < 13$