Semplifica le seguenti espressioni:

1. $$(2+\sqrt{3})\sqrt{7-4\sqrt{3}}+(1-\sqrt{2})\sqrt{3+2\sqrt{2}}$$

$$=(2+\sqrt{3})\sqrt{(2-\sqrt{3})^2}+(1-\sqrt{2})\sqrt{(1+\sqrt{2})^2}=$$

$$=(2+\sqrt{3})(2-\sqrt{3})+(1-\sqrt{2})(1+\sqrt{2})=4-3+1-2=\mathbf{0}$$

2. $$\sqrt{72}+\left[\sqrt[4]{(2\sqrt{2}-4)^4}-\sqrt[9]{(1-\sqrt{2})^9}\right]^2-(2\sqrt{3}+3)(2\sqrt{3}-3)$$

$$=6\sqrt{2}+(4-2\sqrt{2}-1+\sqrt{2})^2-(12-9)=$$

$$=6\sqrt{2}+(3-\sqrt{2})^2-3=6\sqrt{2}+9+2-6\sqrt{2}-3=\mathbf{8}$$

3. $$\left(\sqrt{4+\sqrt{12}}-\sqrt{12}\right)(\sqrt{3}-1)+(1-\sqrt{3})^2$$

$$=\left(\sqrt{4+2\sqrt{3}}-2\sqrt{3}\right)(\sqrt{3}-1)+(1-\sqrt{3})^2=\left(\sqrt{(1+\sqrt{3})^2}-2\sqrt{3}\right)(\sqrt{3}-1)+(1-\sqrt{3})^2=$$

$$=(1+\sqrt{3}-2\sqrt{3})(\sqrt{3}-1)+(1-\sqrt{3})^2=-(1-\sqrt{3})^2+(1-\sqrt{3})^2=\mathbf{0}$$

4. $$\left(\sqrt[3]{32}\cdot\sqrt[3]{2}+\sqrt[3]{-56}:\sqrt[3]{7}+\sqrt{3}\right)(2-\sqrt{3})$$

$$=(\sqrt[3]{64}-\sqrt[3]{8}+\sqrt{3})(2-\sqrt{3})=(4-2+\sqrt{3})(2-\sqrt{3})=(2+\sqrt{3})(2-\sqrt{3})=4-3=\mathbf{1}$$

Determina il dominio delle seguenti funzioni:

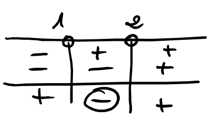
5. $$y=\sqrt{|x+1|-5}+\frac{\sqrt{x-7}}{2}$$

$\begin{cases}|x+1|-5\geq 0\\ x-7\geq 0\end{cases}$ $\quad\begin{cases}x+1\leq -5 \quad \vee \quad x+1\geq 5\\ x\geq 7\end{cases}$ $\quad\begin{cases}x\leq -6 \quad \vee \quad x\geq 4\\ x\geq 7\end{cases}$ $\quad \boldsymbol{x\geq 7}$

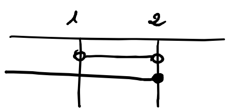
6. $$y=\sqrt{\frac{1}{x-1}-\frac{1}{x-2}}+\sqrt{\frac{2-x}{1+x+x^2}}$$

$\begin{cases}\dfrac{1}{x-1}-\dfrac{1}{x-2}\geq 0\\ \dfrac{2-x}{1+x+x^2}\geq 0\end{cases}$ $\quad\begin{cases}\dfrac{x-2-x+1}{(x-1)(x-2)}\geq 0\\ 2-x\geq 0\end{cases}$ $\quad\begin{cases}\dfrac{-1}{(x-1)(x-2)}\geq 0\\ -x\geq -2\end{cases}$ $\quad\begin{cases}(x-1)(x-2)<0\\ x\leq 2\end{cases}$

$IF>0:\ x>1$
$IIF>0:\ x>2$



$\begin{cases}1<x<2\\ x\leq 2\end{cases}$



$\boldsymbol{1<x<2}$

7. $\begin{cases} x\sqrt{5}+y\sqrt{6}=28 \\ \dfrac{\sqrt{5}}{2}x+\dfrac{2\sqrt{6}}{3}y=17 \end{cases}$

$-\dfrac{\begin{cases} 3x\sqrt{5}+3y\sqrt{6}=84 \\ 3x\sqrt{5}+4y\sqrt{6}=102 \end{cases}}{-y\sqrt{6}=-18} \quad \Rightarrow \quad y=\dfrac{18}{\sqrt{6}}\cdot\dfrac{\sqrt{6}}{\sqrt{6}}=3\sqrt{6} \quad \begin{cases} x\sqrt{5}+18=28 \\ y=3\sqrt{6} \end{cases} \quad \Rightarrow \quad \begin{cases} x=\dfrac{10}{\sqrt{5}}\cdot\dfrac{\sqrt{5}}{\sqrt{5}} \\ y=3\sqrt{6} \end{cases} \quad \begin{cases} \boldsymbol{x=2\sqrt{5}} \\ \boldsymbol{y=3\sqrt{6}} \end{cases}$

8. $\begin{cases} x\sqrt{7}+y\sqrt{10}=0 \\ x+y=\sqrt{7}-\sqrt{10} \end{cases}$

$\begin{cases} x=-y\dfrac{\sqrt{10}}{\sqrt{7}} \\ -\dfrac{y\sqrt{10}}{\sqrt{7}}+y=\sqrt{7}-\sqrt{10} \end{cases} \quad \begin{cases} x=-y\dfrac{\sqrt{10}}{\sqrt{7}} \\ -y\sqrt{10}+y\sqrt{7}=\sqrt{7}(\sqrt{7}-\sqrt{10}) \end{cases} \quad \begin{cases} x=-y\dfrac{\sqrt{10}}{\sqrt{7}} \\ y(\sqrt{7}-\sqrt{10})=\sqrt{7}(\sqrt{7}-\sqrt{10}) \end{cases} \quad \begin{cases} \boldsymbol{x=-\sqrt{10}} \\ \boldsymbol{y=\sqrt{7}} \end{cases}$

9. $\dfrac{x-\sqrt{6}}{\sqrt{3}-1}-\dfrac{x-\sqrt{2}}{\sqrt{3}}+\sqrt{2}=0$

$\dfrac{\sqrt{3}(x-\sqrt{6})-(x-\sqrt{2})(\sqrt{3}-1)+\sqrt{6}(\sqrt{3}-1)}{\sqrt{3}(\sqrt{3}-1)}=0 \quad x\sqrt{3}-3\sqrt{2}-x\sqrt{3}+x+\sqrt{6}-\sqrt{2}+3\sqrt{2}-\sqrt{6}=0 \quad \boldsymbol{x=\sqrt{2}}$

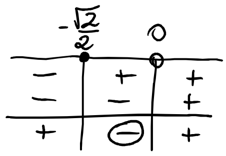
10. $\dfrac{x+\sqrt{3}}{x-\sqrt{3}}-\dfrac{x-\sqrt{3}}{x+\sqrt{3}}=\dfrac{12}{3-x^2}$

$\dfrac{x+\sqrt{3}}{x-\sqrt{3}}-\dfrac{x-\sqrt{3}}{x+\sqrt{3}}=-\dfrac{12}{(x-\sqrt{3})(x+\sqrt{3})} \quad \dfrac{(x+\sqrt{3})^2-(x-\sqrt{3})^2+12}{(x-\sqrt{3})(x+\sqrt{3})}=0 \quad C.A.: x\neq\pm\sqrt{3}$

$x^2+2x\sqrt{3}+3-x^2+2x\sqrt{3}-3+12=0 \quad 4x\sqrt{3}=-12 \quad x=-\frac{3}{\sqrt{3}}=-\sqrt{3} \quad non\ acc. \quad \boldsymbol{\nexists x\in\mathbb{R}}$

11. $\sqrt{2}+\dfrac{1}{x}\leq 0$

$\dfrac{x\sqrt{2}+1}{x}\leq 0 \qquad \begin{array}{ll} N\geq 0 & x\geq -\dfrac{\sqrt{2}}{2} \\ D>0 & x>0 \end{array}$

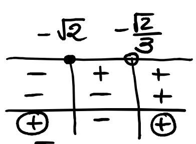


$\boldsymbol{-\dfrac{\sqrt{2}}{2}\leq x<0}$

12. $\begin{cases} \dfrac{2x+\sqrt{8}}{9x+\sqrt{18}}\geq 0 \\ 9x+\sqrt{27}<0 \end{cases}$

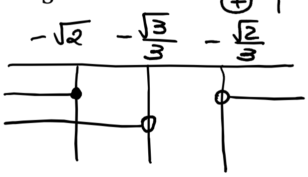
$\begin{cases} \dfrac{2x+2\sqrt{2}}{9x+3\sqrt{2}}\geq 0 \\ 9x+3\sqrt{3}<0 \end{cases} \quad \begin{cases} \dfrac{x+\sqrt{2}}{3x+\sqrt{2}}\geq 0 \\ x<-\dfrac{\sqrt{3}}{3} \end{cases}$

$\dfrac{x+\sqrt{2}}{3x+\sqrt{2}}\geq 0 \qquad \begin{array}{l} N\geq 0: x\geq -\sqrt{2} \\ D>0: x>-\dfrac{\sqrt{2}}{3} \end{array}$



$x\leq -\sqrt{2} \quad \vee \quad x>-\dfrac{\sqrt{2}}{3}$

$\begin{cases} x\leq -\sqrt{2} \quad \vee \quad x>-\dfrac{\sqrt{2}}{3} \\ x<-\dfrac{\sqrt{3}}{3} \end{cases}$



$\boldsymbol{x\leq -\sqrt{2}}$