Semplifica le seguenti espressioni supponendo che i valori delle variabili che in esse figurano soddisfino le condizioni di esistenza:

1. $\frac{[tg(\alpha-\pi)-ctg(\pi-\alpha)]\cdot[ctg(-\alpha)+tg(\pi+\alpha)]}{ctg(-\alpha)-tg(-\alpha)}-\sec\,(2\pi+\alpha)\,cosec\,(11\pi-\alpha)$

$$=\frac{[tg\alpha+ctg\alpha]\cdot[-ctg\alpha+tg\alpha]}{-ctg\alpha+tg\alpha}-\frac{1}{\cos\alpha}\cdot\frac{1}{sen\alpha}=\frac{sen\alpha}{\cos\alpha}+\frac{\cos\alpha}{sen\alpha}-\frac{1}{\cos\alpha\, sen\alpha}=$$

$$=\frac{sen^2\alpha+\cos^2\alpha-1}{\cos\alpha\, sen\alpha}=\frac{1-1}{\cos\alpha\, sen\alpha}=\mathbf{0}$$

2. $\frac{sen\,(360°-\beta)\,ctg\,(-\beta)-\cos\,(180°-\beta)}{\sqrt{2}\,sen\,45°\cos\,(180°+\beta)}-\frac{\cos\,(-\beta)\,tg\,(180°-\beta)+sen\,(\beta-180°)}{2\sqrt{3}\,sen\,60°\,sen\,(-\beta)}$

$$\frac{-sen\,\beta(-ctg\,\beta)+\cos\,\beta}{\sqrt{2}\frac{\sqrt{2}}{2}(-\cos\,\beta)}-\frac{\cos\,\beta\,\,(-tg\,\beta)-\,sen\,\beta}{2\sqrt{3}\frac{\sqrt{3}}{2}\,(-sen\,\beta)}=\frac{sen\beta\frac{\cos\beta}{sen\beta}+\cos\,\beta}{-\cos\,\beta}-\frac{\cos\,\beta\left(-\frac{sen\beta}{\cos\beta}\right)-\,sen\,\beta}{3\,(-sen\,\beta)}=$$

$$=\frac{\cos\,\beta+\cos\,\beta}{-\cos\,\beta}-\frac{-sen\,\beta-\,sen\,\beta}{3\,(-sen\,\beta)}=\frac{2\cos\,\beta}{-\cos\,\beta}-\frac{-2\,sen\,\beta}{3\,(-sen\,\beta)}=-2-\frac{2}{3}=\frac{-6-2}{3}=-\mathbf{\frac{8}{3}}$$

3. $sen^2(4\pi-\beta)+sen^2\left(\frac{13}{2}\pi+\beta\right)+2\,tg\,(-\pi-\beta)\,ctg\,(-3\pi+\beta)+3\cos\,(-\beta)\,cosec\left(\frac{5}{2}\pi+\beta\right)$

$$(-sen\,\beta)^2+(\cos\beta)^2+2\,(-tg\,\beta)\,ctg\,\beta+3\cos\,\beta\frac{1}{\cos\beta}=1-2+3=\mathbf{-2}$$

4. $\frac{[\sqrt{3}\,tg30°\,cosec\,(90°-\alpha)][tg\,45°\,sen\,(270°+\alpha)]}{\sqrt{2}\,sen\,45°\cos\,(180°+\alpha)+\sqrt{3}\,tg\,60°\,sen\,(90°+\alpha)}$

$$=\frac{\left[\sqrt{3}\frac{\sqrt{3}}{3}\frac{1}{\cos\alpha}\right][1\,(-\cos\alpha)]}{\sqrt{2}\frac{\sqrt{2}}{2}(-\cos\,\alpha)+\sqrt{3}\sqrt{3}\cos\alpha}=\frac{\frac{1}{\cos\alpha}(-\cos\alpha)}{-\cos\,\alpha+3\cos\alpha}=\frac{-1}{2\cos\alpha}=-\frac{\mathbf{1}}{\mathbf{2}}\boldsymbol{sec\,\alpha}$$

5. $\frac{2}{tg\,(-x-5\pi)\,cosec\,(x+3\pi)}+\frac{2}{sen\left(\frac{3}{2}\pi-x\right)cosec^2\left(x+\frac{7}{2}\pi\right)}-\,sen\,(-x-13\pi)\cos\,(13\pi+x)$

$$=\frac{2}{-tg\,x\,\frac{1}{-senx}}+\frac{2}{-\cos x\frac{1}{(-\cos x)^2}}-\,sen\,x\,(-\cos\,x)=\frac{2}{-\frac{senx}{\cos x}\frac{1}{-senx}}+\frac{2}{-\cos x\frac{1}{\cos^2 x}}+\,sen\,x\cos\,x=$$

$$=2\cos x-2\cos x+\,sen\,x\cos x=\boldsymbol{sen\,x\cos x}$$

Calcola il valore delle seguenti espressioni:

6. $\sqrt{2}\cos\,(-45°) + 2\sqrt{3}\,sen\,120° - \sqrt{3}\,tg\,60° - 3\,tg\,210°$

$$= \sqrt{2}\frac{\sqrt{2}}{2} + 2\sqrt{3}\frac{\sqrt{3}}{2} - \sqrt{3}\,\sqrt{3} - 3\frac{\sqrt{3}}{3} = 1 + 3 - 3 - \sqrt{3} = \mathbf{1 - \sqrt{3}}$$

7. $sen\frac{5}{2}\pi\,cosec\frac{25}{6}\pi + \cos\frac{3}{2}\pi\;cosec\frac{\pi}{2} + tg\frac{3}{4}\pi\,sen\frac{5}{6}\pi$

$$= 1 \cdot \frac{1}{\frac{1}{2}} + 0 + (-1)\cdot\frac{1}{2} = 2 - \frac{1}{2} = \frac{4-1}{2} = \mathbf{\frac{3}{2}}$$

8. $-2\sqrt{3}\,sen\frac{5}{3}\pi - 3\sqrt{2}\cos\left(-\frac{7}{4}\pi\right) + tg^2\frac{\pi}{3} - ctg^2\left(-\frac{13}{6}\pi\right) - 2\sec\frac{3}{4}\pi\,\left(sen\frac{\pi}{2} - \cos\frac{11}{3}\pi\right)$

$$= -2\sqrt{3}\left(-\frac{\sqrt{3}}{2}\right) - 3\sqrt{2}\cdot\frac{\sqrt{2}}{2} + \left(\sqrt{3}\right)^2 - \left(-\sqrt{3}\right)^2 - 2\frac{1}{-\frac{\sqrt{2}}{2}}\left(1 - \frac{1}{2}\right) = 3 - 3 + 3 - 3 + \frac{4}{\sqrt{2}}\cdot\frac{1}{2} = \mathbf{\sqrt{2}}$$

9. $\dfrac{2\,(x+y)^2\cos\frac{\pi}{3} - 8xy\cos^2\frac{5}{4}\pi}{(x-y)^2\;tg^2\frac{7}{4}\pi - \sqrt{3}\,xy\,ctg\frac{4}{3}\pi - y^2\,sen\left(-\frac{5}{2}\pi\right)}$

$$= \frac{2\frac{(x+y)^2}{2} - 8xy\left(-\frac{\sqrt{2}}{2}\right)^2}{(x-y)^2\; - \sqrt{3}\,xy\,\left(\frac{\sqrt{3}}{3}\right) - y^2\,(-1)} = \frac{x^2+y^2+2xy-4xy}{x^2+y^2-2xy-xy+y^2} = \frac{(x-y)^2}{(x-y)(x-2y)} = \mathbf{\frac{x-y}{x-2y}}$$

Verifica le seguenti identità supponendo che le variabili assumano valori per i quali le espressioni in esse contenute abbiano significato:

10. $cos\ (\alpha+\beta)\, cos\ (\alpha-\beta) = 1 - sen^2\alpha - sen^2\beta$

$$(cos\ \alpha \cos\beta - sen\,\alpha\, sen\,\beta)\,(cos\ \alpha\cos\beta + sen\,\alpha\, sen\,\beta)\ = 1 - sen^2\alpha - sen^2\beta$$

$$cos^2\,\alpha \cos^2\beta - sen^2\,\alpha\, sen^2\,\beta\ = 1 - sen^2\alpha - sen^2\beta$$

$$(1-sen^2\alpha)(1-sen^2\beta) - sen^2\,\alpha\, sen^2\,\beta\ = 1 - sen^2\alpha - sen^2\beta$$

$$1 - sen^2\,\beta - sen^2\alpha + sen^2\,\alpha\, sen^2\,\beta - sen^2\,\alpha\, sen^2\,\beta\ = 1 - sen^2\alpha - sen^2\beta$$

$$\mathbf{1 - sen^2\,\beta - sen^2\alpha = 1 - sen^2\alpha - sen^2\beta}$$

11. $sen\ (30° + \alpha) + \cos\ (30° - \alpha) = \frac{1+\sqrt{3}}{2}\ (sen\ \alpha + \cos\ \alpha)$

$$sen\ 30° \cos\alpha + \cos 30°\ \ sen\ \alpha\ + \cos\ 30° \cos\alpha + sen\ 30°\ sen\ \alpha = \frac{1+\sqrt{3}}{2}\ (sen\ \alpha + \cos\ \alpha)$$

$$\frac{1}{2}\cos\alpha + \frac{\sqrt{3}}{2}\ sen\ \alpha\ + \frac{\sqrt{3}}{2}\cos\alpha + \frac{1}{2}\ sen\ \alpha = \frac{1+\sqrt{3}}{2}\ \ sen\ \alpha + \frac{1+\sqrt{3}}{2}\ \cos\alpha$$

$$\mathbf{\frac{1+\sqrt{3}}{2}\ \ sen\ \alpha + \frac{1+\sqrt{3}}{2}\ \cos\alpha = \frac{1+\sqrt{3}}{2}\ \ sen\ \alpha + \frac{1+\sqrt{3}}{2}\ \cos\alpha}$$

12. $\frac{2\cos\ 2\alpha + sen\ 4\alpha}{2\cos\ 2\alpha - sen\ 4\alpha} = \frac{\sec\ 2\alpha + tg\ 2\alpha}{\sec\ 2\alpha - tg\ 2\alpha}$

$$\frac{2\cos\ 2\alpha\ + 2\ sen\ 2\alpha\ \cos\ 2\alpha}{2\cos\ 2\alpha\ - 2\ sen\ 2\alpha\ \cos\ 2\alpha} = \frac{\frac{1}{\cos 2\alpha} + \frac{sen\ 2\alpha}{\cos 2\alpha}}{\frac{1}{\cos 2\alpha} - \frac{sen\ 2\alpha}{\cos 2\alpha}}$$

$$\frac{2\cos\ 2\alpha\ (1+\ sen\ 2\alpha\,)}{2\cos\ 2\alpha\ (1-\ sen\ 2\alpha\,)} = \frac{\frac{1+\ sen\ 2\alpha}{\cos 2\alpha}}{\frac{1-\ sen\ 2\alpha}{\cos 2\alpha}}$$

$$\mathbf{\frac{1+\ sen\ 2\alpha}{1-\ sen\ 2\alpha} = \frac{1+\ sen\ 2\alpha}{1-\ sen\ 2\alpha}}$$

13. $\frac{\cos\ 2\alpha}{\cos\ \alpha + sen\ \alpha} + \frac{sen\ 2\alpha}{\cos\ \alpha - sen\ \alpha} = \frac{1}{\sqrt{2}\cos\ (45°+\alpha)}$

$$\frac{\cos^2\alpha - sen^2\alpha}{\cos\ \alpha\ +\ sen\ \alpha} + \frac{2\ sen\ \alpha\cos\alpha}{\cos\ \alpha\ -\ sen\ \alpha} = \frac{1}{\sqrt{2}\,(\cos 45°\cos\alpha - sen\ 45°\ sen\ \alpha)}$$

$$\frac{(\cos\ \alpha\ +\ sen\ \alpha)(\cos\ \alpha\ -\ sen\ \alpha)}{\cos\ \alpha\ +\ sen\ \alpha} + \frac{2\ sen\ \alpha\cos\alpha}{\cos\ \alpha\ -\ sen\ \alpha} = \frac{1}{\sqrt{2}\left(\frac{\sqrt{2}}{2}\cos\alpha - \frac{\sqrt{2}}{2}\ sen\ \alpha\right)}$$

$$\cos\ \alpha\ -\ sen\ \alpha + \frac{2\ sen\ \alpha\cos\alpha}{\cos\ \alpha\ -\ sen\ \alpha} = \frac{1}{\cos\ \alpha\ -\ sen\ \alpha}$$

$$\cos^2\alpha +\ sen^2\,\alpha - 2sen\ \alpha\cos\alpha + 2sen\ \alpha\cos\alpha = 1$$

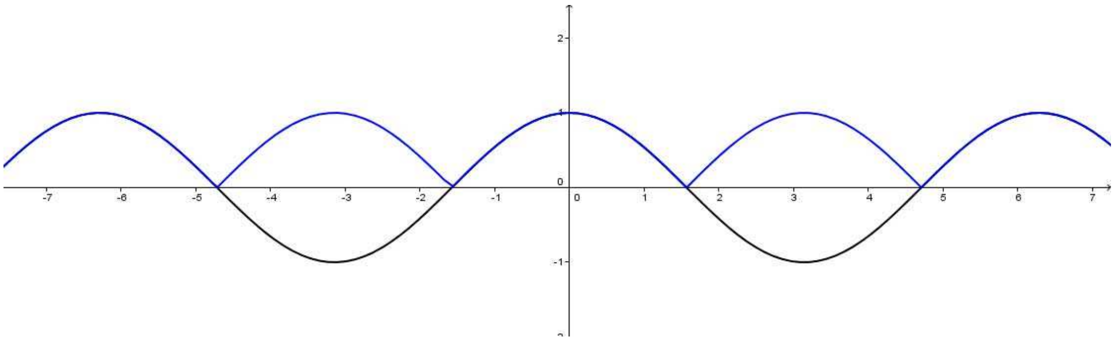
$$\cos^2\alpha +\ sen^2\,\alpha = 1 \qquad \Rightarrow \qquad \mathbf{1 = 1}$$

Traccia il grafico delle seguenti funzioni:

14. $y = |\cos\ x|$ $\quad y = sen\left(x - \frac{\pi}{6}\right)$ $\quad y = \left|tg\left(x + \frac{\pi}{4}\right)\right|$ $\quad y = 3\cos\ 2x$ $\quad y = sen\left(2x + \frac{\pi}{3}\right)$

$y = \cos\ x$

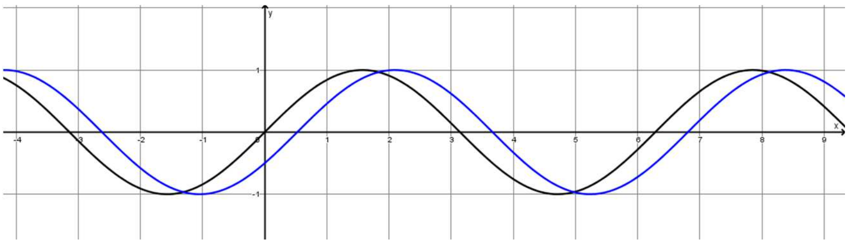
$y = |\cos\ x|$



$y = sen\, x$

Traslazione secondo il vettore $\vec{v}\left(\frac{\pi}{6}; 0\right)$

$y = sen\left(x - \frac{\pi}{6}\right)$

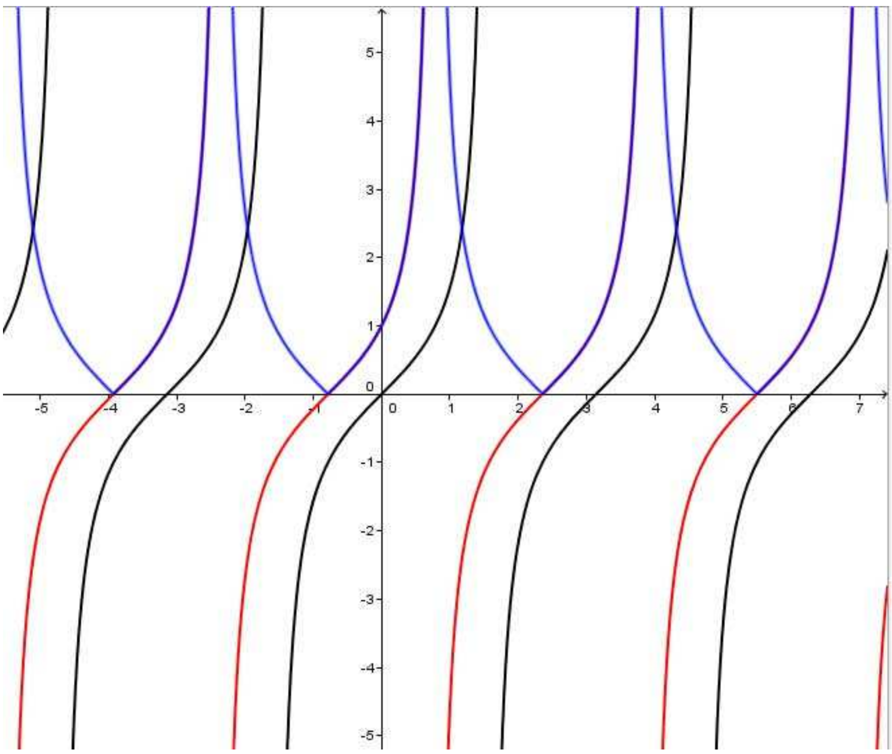


$y = tg\, x$

Traslazione secondo il vettore $\vec{v}\left(-\frac{\pi}{4}; 0\right)$

$y = tg\left(x + \frac{\pi}{4}\right)$

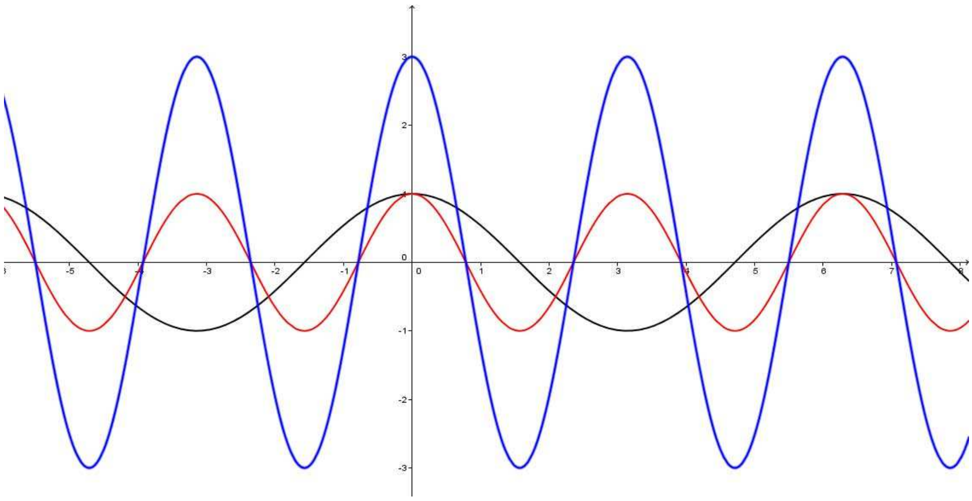
$y = \left|tg\left(x + \frac{\pi}{4}\right)\right|$



$y = \cos\ x$

$y = \cos\ 2x$

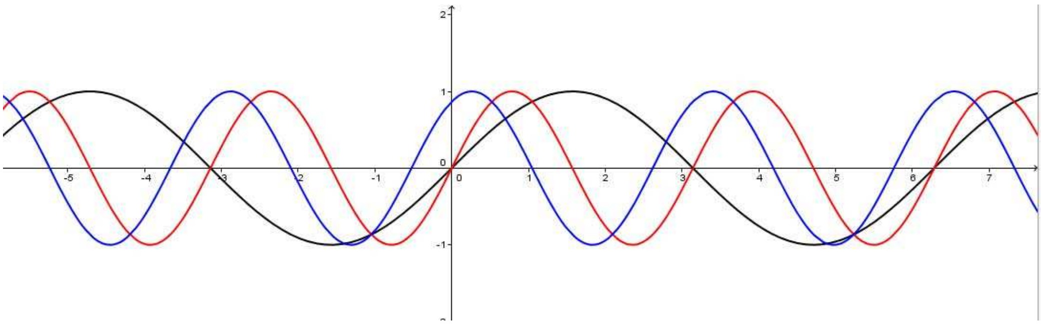
$y = 3\cos\ 2x$



$y = sen\, x$

$y = sen\, (2x)$

$y = sen\left(2x + \frac{\pi}{3}\right)$

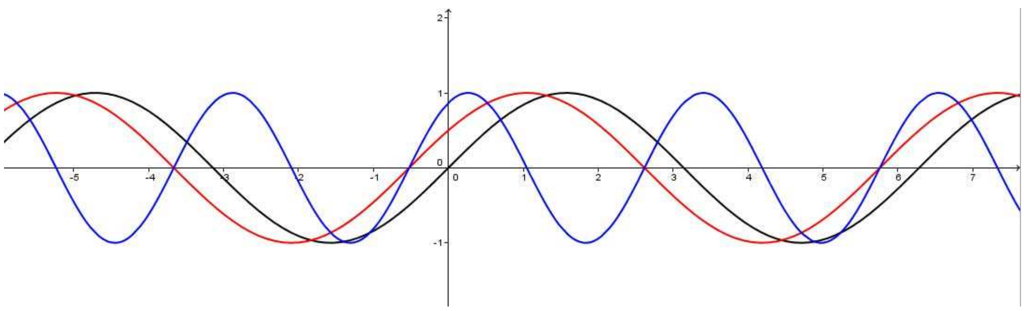


Oppure:

$y = sen\, x$

$y = sen\left(x + \frac{\pi}{6}\right)$

$y = sen\left(2x + \frac{\pi}{3}\right)$



Determina il dominio delle seguenti funzioni:

15. $y = arc\cos\frac{x+1}{x-2}$

$$-1 \leq \frac{x+1}{x-2} \leq 1$$

$$\begin{cases} \dfrac{x+1}{x-2} \leq 1 \\ \dfrac{x+1}{x-2} \geq -1 \end{cases} \qquad \begin{cases} \dfrac{3}{x-2} \leq 0 \\ \dfrac{2x-1}{x-2} \geq 0 \end{cases} \qquad \begin{cases} x < 2 \\ x \leq \dfrac{1}{2} \quad \vee \quad x > 2 \end{cases} \qquad \boldsymbol{x \leq \frac{1}{2}}$$

16. $y = arc\ tg\ \sqrt{3x-2}$

$$3x - 2 \geq 0 \quad \Longrightarrow \quad \boldsymbol{x \geq \frac{2}{3}}$$

17. $y = ctg\ (x - \pi)$

$$x - \pi \neq 0 + k\pi \quad \Longrightarrow \quad \boldsymbol{x \neq \pi + k\pi}$$