Semplifica le seguenti espressioni supponendo che i valori delle variabili che in esse figurano soddisfino le condizioni di esistenza:

1. $\frac{cosec\,(180°-\beta)\sec\,(360°-\beta)}{tg\,(720°+\beta)\,ctg\,(-180°+\beta)}\cdot\frac{sen^2\,(720°-\beta)\cos^2(-180°-\beta)}{2\,tg\,180°-3\,ctg\,45°}+\frac{1}{\sec\,(90°+\beta)\;cosec\,(270°-\beta)}$

$$=\frac{\frac{1}{sen\,\beta}\cdot\frac{1}{\cos\beta}}{tg\,\beta\;ctg\,\beta}\cdot\frac{sen^2\,\beta\cos^2\beta}{2\cdot 0-3\cdot 1}+\frac{1}{\frac{1}{-sen\beta}\cdot\frac{1}{-\cos\beta}}=\frac{\frac{1}{sen\,\beta\cos\beta}}{1}\cdot\frac{sen^2\,\beta\cos^2\beta}{-3}+\frac{sen\,\beta\cos\beta}{1}=$$

$$=-\frac{1}{3}sen\,\beta\cos\beta+sen\,\beta\cos\beta=\mathbf{\frac{2}{3}sen\,\beta\cos\beta}$$

2. $-2\,sen^2\,(-\alpha)\,tg\,\left(\frac{7}{2}\pi+\alpha\right)\,ctg\,(\pi-\alpha)-tg\,(-\alpha)cosec\,(-6\pi+\alpha)-2sen\,(15\pi+\alpha)\cos\,\left(\frac{15}{2}\pi-\alpha\right)$

$$=-2\,sen^2\,\alpha\,(-ctg\,\alpha)(-ctg\,\alpha)-(-tg\,\alpha)\frac{1}{sen\,\alpha}-2\,(-sen\,\alpha)(-sen\,\alpha)=$$

$$=-2\,sen^2\,\alpha\cdot\frac{\cos\alpha}{sen\,\alpha}\cdot\frac{\cos\alpha}{sen\,\alpha}+\frac{sen\,\alpha}{\cos\alpha}\cdot\frac{1}{sen\,\alpha}-2\,sen^2\,\alpha=-2\cos^2\alpha+\frac{1}{\cos\alpha}-2\,sen^2\alpha=$$

$$=-2\,(\cos^2\alpha+sen^2\alpha)+\frac{1}{\cos\alpha}=\mathbf{\sec\,\alpha-2}$$

3. $\left[\cos\,\left(\frac{3}{2}\pi+\alpha\right)+sen\,\left(\alpha-\frac{3}{2}\pi\right)\right]^2-2sen^2\,(-\alpha)\,sen\,\left(-\alpha+\frac{\pi}{2}\right)\,tg\left(\alpha-\frac{\pi}{2}\right)\,cosec\,\left(\alpha-\frac{\pi}{2}\right)$

$$=[sen\,\alpha+\cos\alpha]^2-2\,sen^2\,\alpha\cos\alpha\,(-ctg\,\alpha)\frac{1}{-\cos\alpha}=$$

$$=sen^2\alpha+\cos^2\alpha+2\,sen\alpha\cos\alpha-2\,sen^2\,\alpha\cos\alpha\left(-\frac{\cos\alpha}{sen\,\alpha}\right)\frac{1}{-\cos\alpha}=$$

$$=1+2\,sen\alpha\cos\alpha-2\,sen\alpha\cos\alpha=\mathbf{1}$$

Calcola il valore delle seguenti espressioni:

4. $\sqrt{\frac{1-sen\,120°}{1-sen\,300°}}+\sqrt{\frac{1+\cos\,(-60°)}{1+\cos\,120°}}$

$$=\sqrt{\frac{1-\frac{\sqrt{3}}{2}}{1+\frac{\sqrt{3}}{2}}}+\sqrt{\frac{1+\frac{1}{2}}{1-\frac{1}{2}}}=\sqrt{\frac{2-\sqrt{3}}{2+\sqrt{3}}}+\sqrt{\frac{3}{1}}=\sqrt{\frac{2-\sqrt{3}}{2+\sqrt{3}}\cdot\frac{2-\sqrt{3}}{2-\sqrt{3}}}+\sqrt{3}=\frac{2-\sqrt{3}}{1}+\sqrt{3}=2-\sqrt{3}+\sqrt{3}=\mathbf{2}$$

5. $\left(3a\cos\frac{7}{3}\pi+3b\,sen\frac{13}{6}\pi\right)^2-\left(a\,tg\frac{5}{4}\pi-b\,ctg\frac{15}{4}\pi\right)^2-\left(a\,sec\frac{2}{3}\pi+b\,cosec\frac{11}{6}\pi\right)^2$

$$=\left(3a\cdot\frac{1}{2}+3b\cdot\frac{1}{2}\right)^2-(a+b)^2-(-2a-2b)^2=$$

$$=\frac{9}{4}a^2+\frac{9}{4}b^2+\frac{9}{2}ab-a^2-b^2-2ab-4a^2-4b^2-8ab=$$

$$=-\frac{11}{4}a^2-\frac{11}{4}b^2-\frac{11}{2}ab=-\frac{11}{4}\left(a^2+b^2+2ab\right)=-\frac{\mathbf{11}}{\mathbf{4}}(\boldsymbol{a}+\boldsymbol{b})^{\mathbf{2}}$$

Verifica le seguenti identità supponendo che le variabili assumano valori per i quali le espressioni in esse contenute abbiano significato:

6. $sen\alpha+sen\,(\alpha-120°)+sen\,(\alpha-240°)=0$

$$sen\alpha+sen\,\alpha\cos 120°-\cos\alpha\;sen\,120°+sen\,\alpha\cos 240°-\cos\alpha\;sen\,240°=0$$

$$sen\alpha-\frac{1}{2}sen\,\alpha-\frac{\sqrt{3}}{2}\cos\alpha-\frac{1}{2}sen\,\alpha+\frac{\sqrt{3}}{2}\cos\alpha=0$$

$$sen\alpha-sen\,\alpha=0$$

$$\mathbf{0=0}$$

7. $\frac{1}{2}(tg^2\alpha+ctg^2\alpha)=\frac{3+\cos 4\alpha}{1-\cos 4\alpha}$

$$\frac{1}{2}\left(\frac{sen^2\alpha}{\cos^2\alpha}+\frac{\cos^2\alpha}{sen^2\alpha}\right)=\frac{3+\cos^2 2\alpha-sen^2\,2\alpha}{1-\cos^2 2\alpha+sen^2\,2\alpha}$$

$$\frac{sen^4\alpha+\cos^4\alpha}{2\,sen^2\alpha\cos^2\alpha}=\frac{3+\cos^2 2\alpha-1+\cos^2 2\alpha}{sen^2\,2\alpha+sen^2\,2\alpha}$$

$$\frac{sen^4\alpha+\cos^4\alpha}{2\,sen^2\alpha\cos^2\alpha}=\frac{2\,(1+\cos^2 2\alpha)}{2\,sen^2\,2\alpha}$$

$$\frac{sen^4\alpha+\cos^4\alpha}{2\,sen^2\alpha\cos^2\alpha}=\frac{1+\cos^2 2\alpha}{sen^2\,2\alpha}$$

$$\frac{sen^4\alpha+\cos^4\alpha}{2\,sen^2\alpha\cos^2\alpha}=\frac{1+(\cos^2\alpha-sen^2\alpha)^2}{(2\,sen\,\alpha\cos\alpha)^2}$$

$$\frac{sen^4\alpha+\cos^4\alpha}{2\,sen^2\alpha\cos^2\alpha}=\frac{1+sen^4\alpha+\cos^4\alpha-2\,sen^2\alpha\cos^2\alpha}{4\,sen^2\alpha\cos^2\alpha}$$

$$2\,(sen^4\alpha+\cos^4\alpha)=1+sen^4\alpha+\cos^4\alpha-2\,sen^2\alpha\cos^2\alpha$$

$$sen^4\alpha+\cos^4\alpha+2\,sen^2\alpha\cos^2\alpha=1$$

$$(sen^2\alpha+\cos^2\alpha)^2=1$$

$$\mathbf{(1)^2=1}$$

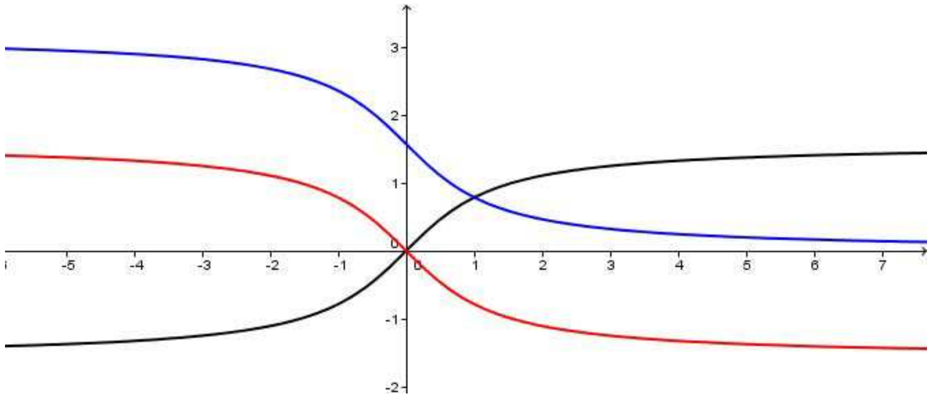
Traccia il grafico delle seguenti funzioni:

8. $y = \text{arc tg }(-x) + \frac{\pi}{2}$ $\quad y = ctg\left(x - \frac{\pi}{6}\right) - 2$ $\quad y = \frac{1}{3}\, sec\ 2x$

$y = \text{arc tg } x$

$y = \text{arc tg }(-x)$

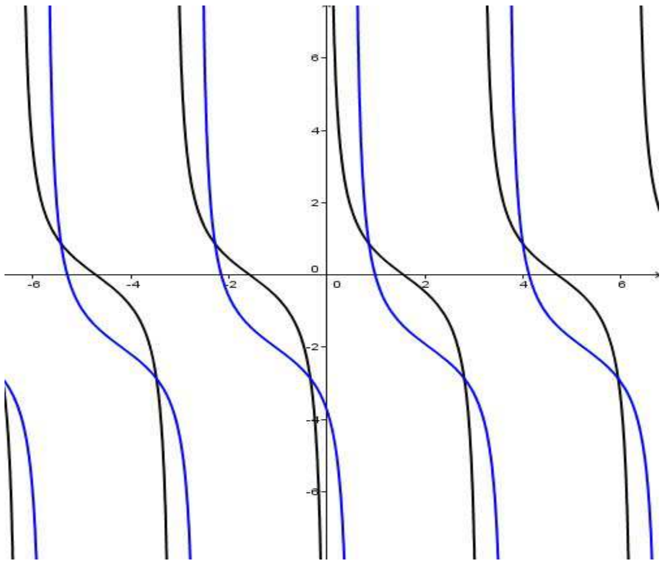
$y = \text{arc tg }(-x) + \frac{\pi}{2}$



$y = ctg\ x$

Traslazione secondo il vettore $\vec{v}\left(\frac{\pi}{6}; -2\right)$

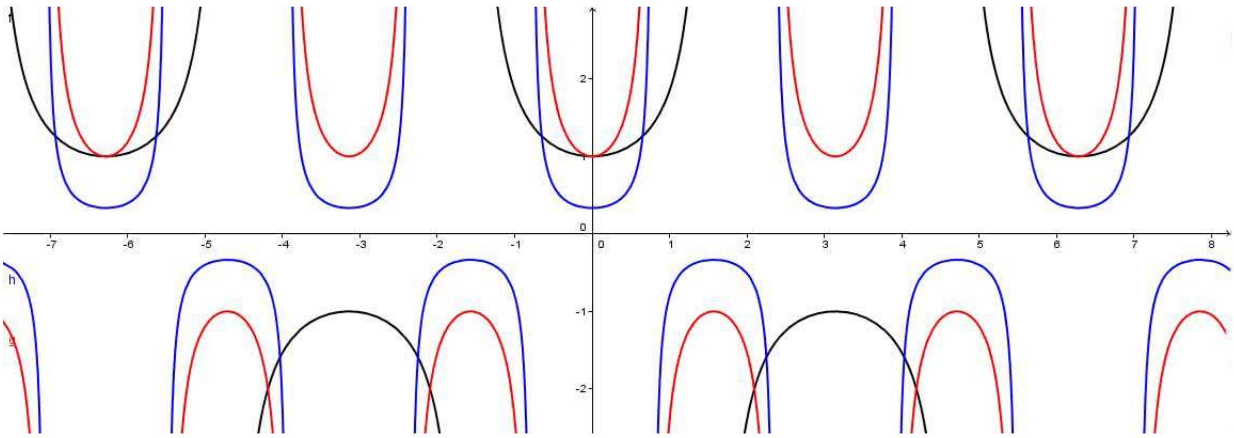
$y = ctg\left(x - \frac{\pi}{6}\right) - 2$



$y = sec\ x$

$y = \sec 2x$

$y = \frac{1}{3}\, sec\ 2x$



Determina il dominio delle seguenti funzioni:

9. $y = arc\,\mathrm{tg}\frac{x+1}{x-2}$ $\quad$ $y = tg\,(x-\pi)$ $\quad$ $y = arc\cos\,(2x-3)$

$y = arc\,\mathrm{tg}\frac{x+1}{x-2}$ $\quad$ $\boldsymbol{x \neq 2}$

$y = tg\,(x-\pi)$ $\quad$ $x - \pi \neq \frac{\pi}{2} + k\pi$ $\quad$ $\boldsymbol{x \neq \frac{3}{2}\pi + k\pi}$

$y = arc\cos\,(2x-3)$ $\quad$ $-1 \leq 2x - 3 \leq 1$ $\quad$ $2 \leq 2x \leq 4$ $\quad$ $\boldsymbol{1 \leq x \leq 2}$