Risolvi le seguenti equazioni:

1. $\dfrac{2}{3}x - \dfrac{2}{3}x\left\{\dfrac{3}{2}x - \dfrac{4}{3}\left[x - \dfrac{1}{6}\left(\dfrac{2}{3}x - \dfrac{3}{2}\right)^2 + \dfrac{2}{3}x\left(\dfrac{1}{9}x - 2\right)\right]\right\} = \dfrac{1}{36}$

$\dfrac{2}{3}x - \dfrac{2}{3}x\left\{\dfrac{3}{2}x - \dfrac{4}{3}\left[x - \dfrac{1}{6}\left(\dfrac{4}{9}x^2 - 2x + \dfrac{9}{4}\right) + \dfrac{2}{27}x^2 - \dfrac{4}{3}x\right]\right\} = \dfrac{1}{36}$

$\dfrac{2}{3}x - \dfrac{2}{3}x\left[\dfrac{3}{2}x - \dfrac{4}{3}\left(x - \dfrac{2}{27}x^2 + \dfrac{1}{3}x - \dfrac{3}{8} + \dfrac{2}{27}x^2 - \dfrac{4}{3}x\right)\right] = \dfrac{1}{36}$

$2x - 2x\left[\dfrac{3}{2}x - \dfrac{4}{3}\left(-\dfrac{3}{8}\right)\right] = \dfrac{1}{12}$ $2x - 2x\left(\dfrac{3}{2}x + \dfrac{1}{2}\right) = \dfrac{1}{12}$ $2x - 3x^2 - x - \dfrac{1}{12} = 0$

$3x^2 - x + \dfrac{1}{12} = 0$ $36x^2 - 12x + 1 = 0$ $(6x - 1)^2 = 0$ $x = \dfrac{\mathbf{1}}{\mathbf{6}}$

2. $\dfrac{2(x-1)(x+1) - x\sqrt{2}}{x\sqrt{2} - 2} = \dfrac{2x^2 - 2}{x\sqrt{2}}$

$\dfrac{2(x^2 - 1) - x\sqrt{2}}{\sqrt{2}(x - \sqrt{2})} = \dfrac{2x^2 - 2}{x\sqrt{2}}$ $\dfrac{2x(x^2 - 1) - x^2\sqrt{2} - (2x^2 - 2)(x - \sqrt{2})}{x\sqrt{2}(x - \sqrt{2})} = 0$ $C.A.:\begin{cases} x \neq 0 \\ x \neq \sqrt{2} \end{cases}$

$2x^3 - 2x - x^2\sqrt{2} - 2x^3 + 2x^2\sqrt{2} + 2x - 2\sqrt{2} = 0$

$x^2\sqrt{2} = 2\sqrt{2}$ $x^2 = 2$ $x_1 = \sqrt{2}\ non\ acc.$ $x_2 = \mathbf{-\sqrt{2}}\ acc.$

3. $\sqrt{2}\left(x + \dfrac{5}{x}\right) + 3 = 3\left(1 - \dfrac{\sqrt{2}}{x}\right)$

$x\sqrt{2} + \dfrac{5\sqrt{2}}{x} + 3 = 3 - \dfrac{3\sqrt{2}}{x}$ $x\sqrt{2} + \dfrac{8\sqrt{2}}{x} = 0$ $x + \dfrac{8}{x} = 0$ $\dfrac{x^2 + 8}{x} = 0$ $\nexists\, \boldsymbol{x} \in \mathbb{R}$

4. $\dfrac{x}{11x + \sqrt{11}} + \dfrac{1}{11} = \dfrac{x^2 + 11}{11x\sqrt{11} + 11}$

$\dfrac{x}{\sqrt{11}(x\sqrt{11} + 1)} + \dfrac{1}{\left(\sqrt{11}\right)^2} = \dfrac{x^2 + 11}{11(x\sqrt{11} + 1)}$ $\dfrac{x\sqrt{11} + x\sqrt{11} + 1 - x^2 - 11}{11(x\sqrt{11} + 1)} = 0$ $C.A.:\ x \neq -\dfrac{\sqrt{11}}{11}$

$x^2 - 2x\sqrt{11} + 10 = 0$ $x_{1,2} = \sqrt{11} \pm \sqrt{11 - 10}$ $x_{1,2} = \mathbf{\sqrt{11} \pm 1}$ $acc.$

5. $\dfrac{\left(\sqrt{5} - 1\right)^2}{x + 2\left(\sqrt{5} - 1\right)} + x = 0$

$\left(\sqrt{5} - 1\right)^2 + x^2 + 2x\left(\sqrt{5} - 1\right) = 0$ $C.A.:\ x \neq 2\left(1 - \sqrt{5}\right)$

$\left[\left(\sqrt{5} - 1\right) + x\right]^2 = 0$ $x_{1,2} = \mathbf{1 - \sqrt{5}}$

Risolvi e discuti le seguenti equazioni letterali:

6. $4k^2x^2 - 4kx + 4k^2x + 1 - 2k = 0$

$4k^2x^2 - 2x(2k - 2k^2) + 1 - 2k = 0$

Se **$k = 0$** $1 = 0$ $\nexists x \in \mathbb{R}$

Se **$k \neq 0$** $\dfrac{\Delta}{4} = (2k - 2k^2)^2 - 4k^2(1 - 2k) = 4k^2 - 8k^3 + 4k^4 - 4k^2 + 8k^3 = 4k^4$

$$x_{1,2} = \frac{2k - 2k^2 \pm 2k^2}{4k^2} = \begin{cases} \dfrac{\mathbf{1}}{\mathbf{2k}} \\ \dfrac{2k(1 - 2k)}{4k^2} = \dfrac{\mathbf{1 - 2k}}{\mathbf{2k}} \end{cases}$$

7. $k^2x^2 - 121\,k^3 = 0$

$k^2(x^2 - 121\,k) = 0$

Se **$k = 0$** $\forall x \in \mathbb{R}$

Se $k \neq 0$ $x^2 = 121\,k$

$Se\ k > 0$ $x_{1,2} = \pm 11\sqrt{k}$ **$Se\ k < 0$** $\nexists x \in \mathbb{R}$

8. $kx(x + 3) - x(x + 3k) = 9k - 9$

$kx^2 + 3kx - x^2 - 3kx = 9k - 9$ $x^2(k - 1) = 9(k - 1)$

Se **$k = 1$** $\forall x \in \mathbb{R}$

Se **$k \neq 1$** $x^2 = 9$ $x_{1,2} = \pm 3$

Data l'equazione parametrica: $x^2 - 2(k - 1)\,x + k^2 - 1 = 0$, stabilisci quale valore deve assumere k perché:

le soluzioni siano reali e distinte: $\frac{\Delta}{4} > 0$: $(k-1)^2 - k^2 + 1 = k^2 - 2k + 1 - k^2 + 1 > 0$ **$k < 1$**

le soluzioni siano coincidenti: $\frac{\Delta}{4} = 0$: **$k = 1$**

le soluzioni siano opposte: $S = 0 \Rightarrow -\frac{b}{a} = 0 \Rightarrow b = 0 \Rightarrow k - 1 = 0 \Rightarrow$ **$k = 1$**

una delle soluzioni sia nulla: $k^2 - 1 = 0 \Rightarrow$ **$k = \pm 1$**

la somma dei reciproci delle soluzioni sia uguale a 2: $\frac{1}{x_1} + \frac{1}{x_2} = 2 \Rightarrow \frac{x_1 + x_2}{x_1 x_2} = 2 \Rightarrow \frac{2(k-1)}{k^2 - 1} = 2 \Rightarrow \frac{1}{k+1} = 1 \Rightarrow$ **$k = 0$**