1. $\int \left(\frac{2}{x^3} + \frac{x^3 - 2x}{4} \right) dx$

$$2\int x^{-3}\,dx + \frac{1}{4}\int x^3\,dx - \frac{1}{2}\int x\,dx = \frac{2}{-2}x^{-2} + \frac{1}{4}\cdot\frac{1}{4}x^4 - \frac{1}{2}\cdot\frac{1}{2}x^2 + c = \boldsymbol{-\frac{1}{x^2} + \frac{x^4}{16} - \frac{x^2}{4} + c}$$

2. $\int \left(\frac{1}{3}\sqrt[3]{x^2} - \frac{7}{6}\sqrt[6]{x} \right) dx$

$$\frac{1}{3}\int x^{\frac{2}{3}}\,dx - \frac{7}{6}\int x^{\frac{1}{6}}\,dx = \frac{1}{3}\cdot\frac{3}{5}\,x^{\frac{5}{3}} - \frac{7}{6}\cdot\frac{6}{7}\,x^{\frac{7}{6}} + c = \boldsymbol{\frac{x\,\sqrt[3]{x^2}}{5} - x\,\sqrt[6]{x} + c}$$

3. $\int \left(\frac{3}{x} + 5\cos x - 3\,senx \right) dx$

$$3\int \frac{1}{x}\,dx + 5\int \cos x\;dx - 3\int sen\,x\,dx = \boldsymbol{3\ln|x| + 5\,sen\,x + 3\cos x + c}$$

4. $\int \frac{e^{\ln x}}{x}\,dx$

$$t = \ln x \qquad dt = \frac{1}{x}dx \qquad \int e^t\,dt = e^t + c = \boldsymbol{e^{\ln x} + c}$$

5. $\int \frac{x^3 - 3x^2 + 2x}{x-2}\,dx$

$$\int \frac{x\,(x^2 - 3x + 2)}{x-2}dx = \int \frac{x\,(x-2)(x-1)}{x-2}\,dx = \int (x^2 - x)\,dx = \boldsymbol{\frac{x^3}{3} - \frac{x^2}{2} + c}$$

6. $\int \frac{6x^3 - 7x^2 + 4x + 1}{2x-1}\,dx$

$6x^3$	$-7x^2$	$4x$	$+1$	$2x-1$
$-6x^3$	$+3x^2$			$3x^2 - 2x + 1$
	$-4x^2$	$4x$	$+1$	
	$+4x^2$	$-2x$		
		$2x$	$+1$	
		$-2x$	$+1$	
			$+2$	

$$\int \left(3x^2 - 2x + 1 + \frac{2}{2x-1} \right) dx =$$

$$= 3\int x^2\,dx - 2\int x\,dx + \int 1\,dx + \int \frac{2}{2x-1}\,dx =$$

$$= \boldsymbol{x^3 - x^2 + x + \ln|2x-1| + c}$$

7. $\int \frac{6x^2+6}{x^3+3x-2}\,dx$

$$2\int \frac{3x^2+3}{x^3+3x-2}\,dx = \mathbf{2\ln\ |x^3+3x-2|\ +c}$$

8. $\int \frac{1}{x\ln x}\,dx$

$$t=\ln x \qquad dt=\frac{1}{x}dx \qquad \int \frac{1}{t}\,dt = \ln t + c = \boldsymbol{ln\,|ln\,x| + c}$$

9. $\int \frac{e^x}{1+e^{2x}}\,dx$

$$t=e^x \qquad dt=e^x dx \qquad \int \frac{1}{1+t^2}\,dt = arctg\,t + c = \boldsymbol{arctg\,e^x + c}$$

10. $\int \frac{x}{e^x}\,dx$

$$\int x\,e^{-x}\,dx \qquad \begin{matrix} f(x)=x & f'(x)=1 \\ g'(x)=e^{-x} & g(x)=-e^{-x} \end{matrix}$$

$$= -xe^{-x} + \int e^{-x}\,dx = -x\,e^{-x} - e^{-x} + c = \boldsymbol{-\frac{x+1}{e^x} + c}$$

11. $\int \frac{x-5}{x^2-12x+36}\,dx$

$$\int \frac{x-5}{(x-6)^2}\,dx = \int \frac{x-5-1+1}{(x-6)^2}\,dx = \int \frac{x-6}{(x-6)^2}\,dx + \int \frac{1}{(x-6)^2}\,dx = \mathbf{ln\ |x-6| - \frac{1}{x-6} + c}$$

12. $\int \ln\ (2x-3)\ dx$

$$\int (1\cdot \ln\ (2x-3))\,dx \qquad \begin{matrix} f(x)=\ln\ (2x-3) & f'(x)=\frac{2}{2x-3} \\ g'(x)=1 & g(x)=x \end{matrix}$$

$$= x\ln\ (2x-3) - \int \frac{2x}{2x-3}\,dx = x\ln\ (2x-3) - \int \frac{2x-3+3}{2x-3}\,dx = x\ln\ (2x-3) +$$

$$-\int \frac{2x-3}{2x-3}\,dx - \frac{3}{2}\int \frac{2}{2x-3}\,dx = x\ln\ (2x-3) - x - \frac{3}{2}\ln\ (2x-3) = \boldsymbol{\frac{2x-3}{2}\ ln\,(2x-3) - x + c}$$

13. $\int \frac{x}{\sqrt{4x^2+1}}\,dx$

$$\frac{1}{8}\int \frac{8x}{\sqrt{4x^2+1}}\,dx = \frac{1}{8}\int 8x\,(4x^2+1)^{-\frac{1}{2}}\,dx = \frac{1}{8}\cdot 2\,(4x^2+1)^{\frac{1}{2}} + c = \boldsymbol{\frac{\sqrt{4x^2+1}}{4} + c}$$

14. $\int \frac{\cos x + 3}{3x + sen\,x}\,dx$

$$D\,(3x + sen\,x) = 3 + \cos x \qquad \int \frac{\cos x + 3}{3x + sen\,x}\,dx = \boldsymbol{\ln|3x + sen\,x| + c}$$

15. $\int \frac{x^2}{1+x^6}\,dx$

$$t = x^3 \qquad dt = 3x^2 dx \qquad \frac{1}{3}\int \frac{1}{1+t^2}\,dt = \frac{1}{3} arctg\,t + c = \boldsymbol{\frac{1}{3} arctg\,x^3 + c}$$

16. $\int \frac{5}{\sqrt{16-25x^2}}\,dx$

$$\frac{5}{4}\int \frac{1}{\sqrt{1-\frac{25}{16}x^2}}\,dx = \int \frac{\frac{5}{4}}{\sqrt{1-\left(\frac{5}{4}x\right)^2}}\,dx = \boldsymbol{arc\,sen\frac{5}{4}x + c}$$

17. $\int \frac{x+5}{x^3-x^2-x+1}\,dx$

$$\int \frac{x+5}{x^2(x-1)-1\,(x-1)}\,dx = \int \frac{x+5}{(x-1)(x^2-1)}\,dx = \int \frac{x+5}{(x-1)^2(x+1)}\,dx$$

$$\frac{x+5}{x^3-x^2-x+1} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x+1} = \frac{A\,(x^2-1) + B\,(x+1) + C\,(x^2-2x+1)}{x^3-x^2-x+1}$$

$$\begin{cases} A + C = 0 \\ B - 2C = 1 \\ -A + B + C = 5 \end{cases} \qquad \begin{cases} A = -C \\ B = 2C + 1 \\ C + 2C + 1 + C = 5 \end{cases} \qquad \begin{cases} A = -1 \\ B = 3 \\ C = 1 \end{cases}$$

$$= \int \left(-\frac{1}{x-1} + \frac{3}{(x-1)^2} + \frac{1}{x+1}\right) dx = -\ln\,|x-1| - \frac{3}{x-1} + \ln\,|x+1| + c = \boldsymbol{\ln\left|\frac{x+1}{x-1}\right| - \frac{3}{x-1} + c}$$

18. $\int \frac{1}{x-\sqrt{x}}\,dx$

$$t=\sqrt{x} \qquad x=t^2 \qquad dx=2t\,dt$$

$$\int \frac{1}{t^2-t}\,2t\,dt = 2\int \frac{t}{t\,(t-1)}\,dt = 2\int \frac{1}{t-1}\,dt = 2\ln\,|t-1|+c = \boldsymbol{2\,ln\left|\sqrt{x}-1\right|+c}$$