Determina il dominio dei seguenti radicali:

1. $\sqrt{x+2}+\sqrt[4]{x-2}+\sqrt[3]{x}$

$$\begin{cases} x+2\geq 0 \\ x-2\geq 0 \end{cases} \Rightarrow \begin{cases} x\geq -2 \\ x\geq 2 \end{cases} \Rightarrow \boldsymbol{x\geq 2}$$

2. $\sqrt{\frac{x+2}{x-4}}+\sqrt[7]{x-3}+\frac{1}{\sqrt[4]{x^2+4}}$

$$\frac{x+2}{x-4}\geq 0 \qquad \begin{array}{l} N\geq 0:\ x\geq -2 \\ D>0:\ x>4 \end{array}$$

	-2		4	
−		+		+
−		−		+
+		−		+

$$\boldsymbol{x\leq -2} \quad \vee \quad \boldsymbol{x>4}$$

3. $\sqrt{|x-2|-3}+\frac{3x+2}{\sqrt[3]{x-6}}$

$$\begin{cases} |x-2|-3\geq 0 \\ x-6\neq 0 \end{cases} \Rightarrow \begin{cases} x-2\leq -3 \quad \vee \quad x-2\geq 3 \\ x\neq 6 \end{cases}$$

$$\begin{cases} x\leq -1 \quad \vee \quad x\geq 5 \\ x\neq 6 \end{cases}$$

$$\boldsymbol{x\leq -1} \quad \vee \quad \boldsymbol{5\leq x<6} \quad \vee \quad \boldsymbol{x>6}$$

Semplifica le seguenti espressioni numeriche:

4. $(\sqrt{3}+\sqrt{5})^2+(\sqrt{5}-\sqrt{3})(\sqrt{5}+\sqrt{3})-\sqrt{15}\cdot\sqrt{(-2)^2}$

$3+5+2\sqrt{15}+5-3-2\sqrt{15}=\boldsymbol{10}$

5. $(\sqrt{18}+\sqrt{50}):\sqrt{2}-(\sqrt{5}+1)^2+\frac{(\sqrt{5}-1)(\sqrt{5}+1)}{(2-\sqrt{3})(2+\sqrt{3})}$

$\sqrt{9}+\sqrt{25}-(5+1+2\sqrt{5})+\frac{5-1}{4-3}=3+5-5-1-2\sqrt{5}+4=\boldsymbol{6-2\sqrt{5}}$

6. $\left(\sqrt{3}+2\right)^3-\left(\sqrt{3}-2\right)^3$

$\left(\sqrt{3}\right)^3+6\left(\sqrt{3}\right)^2+12\sqrt{3}+8-\left[\left(\sqrt{3}\right)^3-6\left(\sqrt{3}\right)^2+12\sqrt{3}-8\right]=$

$=\left(\sqrt{3}\right)^3+18+12\sqrt{3}+8-\left(\sqrt{3}\right)^3+18-12\sqrt{3}+8=\mathbf{52}$

7. $\frac{\sqrt{15}-\sqrt{5}+\sqrt{3}-1}{1-\sqrt{3}}+\frac{\sqrt{10}+2\sqrt{2}-\sqrt{5}-2}{\sqrt{2}-1}-1$

$=\frac{\sqrt{5}\left(\sqrt{3}-1\right)+1\left(\sqrt{3}-1\right)}{-\left(\sqrt{3}-1\right)}+\frac{\sqrt{2}\left(\sqrt{5}+2\right)-\left(\sqrt{5}+2\right)}{\sqrt{2}-1}-1=$

$=\frac{\left(\sqrt{3}-1\right)\left(\sqrt{5}+1\right)}{-\left(\sqrt{3}-1\right)}+\frac{\left(\sqrt{5}+2\right)\left(\sqrt{2}-1\right)}{\sqrt{2}-1}-1=-\sqrt{5}-1+\sqrt{5}+2-1=\mathbf{0}$

8. $\sqrt[4]{\left(\sqrt{3}+3\right)\left(3-\sqrt{3}\right)+\left(1+\sqrt{3}\right)^2-\left(\sqrt{2\sqrt{3}}\right)^2-\left(\sqrt[3]{-6}\right)^3}$

$\sqrt[4]{9-3+1+3+2\sqrt{3}-2\sqrt{3}+6}=\sqrt[4]{16}=\sqrt[4]{2^4}=\mathbf{2}$

9. $\left(\sqrt[12]{64}+\sqrt[5]{3^5}\right)\left(\sqrt{2}-3\right)+\sqrt[6]{(-7)^6}-\sqrt{\left(1-\sqrt{3}\right)^2}+\sqrt[6]{27}+\sqrt[5]{-1}$

$\left(\sqrt[12]{2^6}+3\right)\left(\sqrt{2}-3\right)+\sqrt[6]{7^6}-\sqrt{\left(\sqrt{3}-1\right)^2}+\sqrt[6]{3^3}-\sqrt[5]{1}=$

$=\left(\sqrt{2}+3\right)\left(\sqrt{2}-3\right)+7-\sqrt{3}+1+\sqrt{3}-1=2-9+7=\mathbf{0}$

10. $\left[21\sqrt{2}:\left(3\sqrt{2}\right)-\frac{5\sqrt[4]{4}}{\sqrt{2}}+\sqrt{5}\right]\left(2-\sqrt[4]{25}\right)+3\cdot\sqrt[8]{\left(\frac{4}{9}\right)^3\cdot\left(\frac{2}{3}\right)^2}$

$\left[\frac{21\sqrt{2}}{3\sqrt{2}}-\frac{5\sqrt[4]{2^2}}{\sqrt{2}}+\sqrt{5}\right]\left(2-\sqrt[4]{5^2}\right)+3\cdot\sqrt[8]{\left(\frac{2}{3}\right)^6\cdot\left(\frac{2}{3}\right)^2}=$

$=\left[7-\frac{5\sqrt{2}}{\sqrt{2}}+\sqrt{5}\right]\left(2-\sqrt{5}\right)+3\cdot\sqrt[8]{\left(\frac{2}{3}\right)^8}=$

$=\left[7-5+\sqrt{5}\right]\left(2-\sqrt{5}\right)+3\cdot\frac{2}{3}=\left(2+\sqrt{5}\right)\left(2-\sqrt{5}\right)+2=4-5+2=\mathbf{1}$

Semplifica le seguenti espressioni letterali:

11. $\sqrt{x-1}\cdot\sqrt[3]{1-x}$

$$C.E.:\ x-1\geq 0 \quad\Longrightarrow\quad x\geq 1$$

$$\sqrt{x-1}\cdot\sqrt[3]{-(x-1)}=-\sqrt{x-1}\cdot\sqrt[3]{x-1}=-\sqrt[6]{(x-1)^3\cdot(x-1)^2}=\boldsymbol{-\sqrt[6]{(x-1)^5}}$$

12. $\sqrt[4]{x^2+4}\cdot\sqrt[3]{x^2-4}\cdot\sqrt[6]{\frac{x-2}{x^2+4}}$

$$C.E.:\ x-2\geq 0 \quad\Longrightarrow\quad x\geq 2$$

L'argomento della radice di indice 3 è sicuramente positivo, considerate le condizioni di esistenza:

$$\sqrt[12]{(x^2+4)^3\cdot(x^2-4)^4\cdot\left(\frac{x-2}{x^2+4}\right)^2}=\sqrt[12]{(x^2+4)^3\cdot(x-2)^4\cdot(x+2)^4\cdot\frac{(x-2)^2}{(x^2+4)^2}}=\boldsymbol{\sqrt[12]{(x^2+4)\cdot(x-2)^6\cdot(x+2)^4}}$$

13. $\sqrt{\left(\sqrt{-x}+2\right)^2}-\sqrt[4]{(x)^2}+\sqrt[9]{(-x)^9}-2$

$$C.E.: -x\geq 0 \quad\Longrightarrow\quad x\leq 0$$

$$\sqrt{-x}+2-\sqrt[4]{(-x)^2}-\sqrt[9]{x^9}-2=\sqrt{-x}+2-\sqrt{-x}-x-2=\boldsymbol{-x}$$

Stabilisci M.C.D. e m.c.m. dei seguenti gruppi di polinomi, dopo averli scomposti:

14. $x^2-2 \qquad x\sqrt{2}-2$

$$x^2-2=(x-\sqrt{2})(x+\sqrt{2}) \qquad x\sqrt{2}-2=\sqrt{2}\,(x-\sqrt{2})$$

$$\boldsymbol{M.C.D.=x-\sqrt{2}} \qquad \boldsymbol{m.c.m.=\sqrt{2}\,(x-\sqrt{2})(x+\sqrt{2})}$$

15. $x^2-2x\sqrt{3}+3 \qquad x^2-3 \qquad 3-x\sqrt{3}$

$$x^2-2x\sqrt{3}+3=(x-\sqrt{3})^2 \qquad x^2-3=(x-\sqrt{3})(x+\sqrt{3}) \qquad 3-x\sqrt{3}=-\sqrt{3}(x-\sqrt{3})$$

$$\boldsymbol{M.C.D.=x-\sqrt{3}} \qquad \boldsymbol{m.c.m.=\sqrt{3}\,(x-\sqrt{3})^2(x+\sqrt{3})}$$