Semplifica le seguenti espressioni numeriche:

1. $\left(\sqrt{7-4\sqrt{3}}+\frac{1}{\sqrt{3}-\sqrt{2}}\right)^{-1}\cdot\sqrt{6+4\sqrt{2}}$

$$=\left(\sqrt{\left(2-\sqrt{3}\right)^2}+\frac{1}{\sqrt{3}-\sqrt{2}}\cdot\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}}\right)^{-1}\cdot\sqrt{\left(\sqrt{2}+2\right)^2}=$$

$$=\left(2-\sqrt{3}+\sqrt{3}+\sqrt{2}\right)^{-1}\cdot\left(\sqrt{2}+2\right)=\frac{1}{\sqrt{2}+2}\cdot\left(\sqrt{2}+2\right)=\mathbf{1}$$

2. $\sqrt{\left(1-\sqrt{2}\right)^2}+\sqrt[3]{-\sqrt[5]{-\sqrt{1}}}+\sqrt{\frac{1}{3}\sqrt{3\sqrt{3\sqrt{3}}}-\frac{1}{3}\sqrt[8]{243\sqrt{243}}}$

$$=\sqrt{2}-1+\sqrt[15]{1}+\sqrt{\frac{1}{3}\sqrt{3\sqrt[4]{3^3}}-\frac{1}{3}\sqrt[8]{3^5\sqrt{3^5}}}=\sqrt{2}-1+1+\sqrt{\frac{1}{3}\sqrt[8]{3^7}-\frac{1}{3}\sqrt[16]{3^{15}}}=\sqrt{2}+\sqrt[16]{\frac{1}{3}}-\frac{1}{3}\sqrt[16]{3^{15}}=$$

$$=\sqrt{2}+\sqrt[16]{\frac{1}{3}}\cdot\frac{\sqrt[16]{3^{15}}}{\sqrt[16]{3^{15}}}-\frac{1}{3}\sqrt[16]{3^{15}}=\sqrt{2}+\frac{1}{3}\sqrt[16]{3^{15}}-\frac{1}{3}\sqrt[16]{3^{15}}=\sqrt{\mathbf{2}}$$

Risolvi:

3. $\frac{\sqrt{2}+x}{\sqrt{2}+1}=\frac{\sqrt{6}-\sqrt{3}}{\sqrt{3}}$

$$\frac{\sqrt{2}+x}{\sqrt{2}+1}=\frac{\sqrt{3}\left(\sqrt{2}-1\right)}{\sqrt{3}}\qquad \sqrt{2}+x=\left(\sqrt{2}-1\right)\cdot\left(\sqrt{2}+1\right)\qquad x=\mathbf{1-\sqrt{2}}$$

4. $\frac{x}{3-\sqrt{5}}+\frac{2-x}{3+\sqrt{5}}+1>-\frac{\sqrt{5}}{2}$

$$\frac{x\left(3+\sqrt{5}\right)+(2-x)\left(3-\sqrt{5}\right)+4}{\left(3-\sqrt{5}\right)\left(3+\sqrt{5}\right)}>-\frac{2\sqrt{5}}{4}\qquad 3x+x\sqrt{5}+6-2\sqrt{5}-3x+x\sqrt{5}+4>-2\sqrt{5}$$

$$2x\sqrt{5}>-10\qquad x>-\frac{5}{\sqrt{5}}\qquad \boldsymbol{x>-\sqrt{5}}$$

5. $(\sqrt{3}-x)(\sqrt{3}+x)-(2\sqrt{7}+x)(x-\sqrt{7})+(2x+1)(x-3)<x\,(1-\sqrt{7})$

$$3-x^2-(2x\sqrt{7}-14+x^2-x\sqrt{7})+2x^2-6x+x-3<x-\ x\sqrt{7}$$

$$3-x^2-x\sqrt{7}+14-x^2+2x^2-6x+x-3<x-\ x\sqrt{7}$$

$$-6x<-14 \qquad\qquad \boldsymbol{x>\frac{7}{3}}$$

6. $\frac{2x+2\sqrt{2}}{\sqrt{2}\,(2x-\sqrt{2})}\geq 0$

$N\geq 0: \quad x\geq -\sqrt{2}$

$D>0: \quad x>\frac{\sqrt{2}}{2}$

$$\boldsymbol{x\leq -\sqrt{2}} \quad \vee \quad \boldsymbol{x>\frac{\sqrt{2}}{2}}$$

7. $(x+1)^2+1\leq 2\,(x+2)$

$$x^2+2x+1+1\leq 2x+4 \qquad\qquad x^2-2\leq 0 \qquad\qquad (x-\sqrt{2})(x+\sqrt{2})\leq 0$$

$$\boldsymbol{-\sqrt{2}\leq x\leq \sqrt{2}}$$

8. $\begin{cases} x\sqrt{2}+y\sqrt{3}=0 \\ x+y=\sqrt{2}-\sqrt{3} \end{cases}$

$$\begin{cases} x=-y\frac{\sqrt{3}}{\sqrt{2}} \\ -y\frac{\sqrt{3}}{\sqrt{2}}+y=\sqrt{2}-\sqrt{3} \end{cases} \qquad \begin{cases} x=-y\frac{\sqrt{3}}{\sqrt{2}} \\ y\left(-\frac{\sqrt{3}}{\sqrt{2}}+1\right)=\sqrt{2}-\sqrt{3} \end{cases} \qquad \begin{cases} x=-y\frac{\sqrt{3}}{\sqrt{2}} \\ y\frac{-\sqrt{3}+\sqrt{2}}{\sqrt{2}}=\sqrt{2}-\sqrt{3} \end{cases}$$

$$\begin{cases} x=-y\frac{\sqrt{3}}{\sqrt{2}} \\ \frac{y}{\sqrt{2}}=1 \end{cases} \qquad \begin{cases} x=-\sqrt{2}\frac{\sqrt{3}}{\sqrt{2}} \\ y=\sqrt{2} \end{cases} \qquad \begin{cases} \boldsymbol{x=-\sqrt{3}} \\ \boldsymbol{y=\sqrt{2}} \end{cases}$$

Semplifica la seguente espressione letterale:

9. $\sqrt[3]{\frac{1}{a-1}\sqrt{1-a}}$

$$C.E.:\ 1-a>0 \quad \Rightarrow \quad a<1$$

$$-\sqrt[3]{\frac{1}{1-a}\sqrt{1-a}}=-\sqrt[3]{\sqrt{\frac{1-a}{(1-a)^2}}}=-\sqrt[6]{\frac{\mathbf{1}}{\mathbf{1-a}}}$$

Considera le seguenti espressioni contenenti radicali:

A. Trasformale in espressioni con esponenti frazionari
B. Semplificale utilizzando le proprietà delle potenze
C. Riscrivi i risultati sotto forma di radicale

10. $\sqrt{\frac{3\sqrt{2}}{2\sqrt{3}}}\cdot\sqrt[3]{\frac{9\sqrt{2}}{4\sqrt{3}}}$

$$=\left(3\cdot 2^{\frac{1}{2}}\cdot 2^{-1}\cdot 3^{-\frac{1}{2}}\right)^{\frac{1}{2}}\cdot\left(3^2\cdot 2^{\frac{1}{2}}\cdot 2^{-2}\cdot 3^{-\frac{1}{2}}\right)^{\frac{1}{3}}=\left(3^{\frac{1}{2}}\cdot 2^{-\frac{1}{2}}\right)^{\frac{1}{2}}\cdot\left(3^{\frac{3}{2}}\cdot 2^{-\frac{3}{2}}\right)^{\frac{1}{3}}=$$

$$=3^{\frac{1}{4}}\cdot 2^{-\frac{1}{4}}\cdot 3^{\frac{1}{2}}\cdot 2^{-\frac{1}{2}}=3^{\frac{3}{4}}\cdot 2^{-\frac{3}{4}}=\left(\frac{3}{2}\right)^{\frac{3}{4}}=\sqrt[4]{\frac{\mathbf{27}}{\mathbf{8}}}$$

11. $\sqrt{\frac{1}{2}\sqrt[3]{2}}\cdot\sqrt[3]{4\sqrt{2}}\cdot\sqrt{\frac{1}{2}}$

$$=\left(2^{-1}\cdot 2^{\frac{1}{3}}\right)^{\frac{1}{2}}\cdot\left(2^2\cdot 2^{\frac{1}{2}}\right)^{\frac{1}{3}}\cdot 2^{-\frac{1}{2}}=\left(2^{-\frac{2}{3}}\right)^{\frac{1}{2}}\cdot\left(2^{\frac{5}{2}}\right)^{\frac{1}{3}}\cdot 2^{-\frac{1}{2}}=2^{-\frac{1}{3}}\cdot 2^{\frac{5}{6}}\cdot 2^{-\frac{1}{2}}=$$

$$=2^{-\frac{1}{3}+\frac{5}{6}-\frac{1}{2}}=2^{\frac{-2+5-3}{6}}=2^0=\mathbf{1}$$