Semplifica le seguenti espressioni numeriche:

1. $\left(\sqrt{11-6\sqrt{2}}+\frac{1}{\sqrt{3}-\sqrt{2}}\right)^{-1}\cdot\sqrt{12+6\sqrt{3}}$

$$=\left(\sqrt{\left(3-\sqrt{2}\right)^2}+\frac{1}{\sqrt{3}-\sqrt{2}}\cdot\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}}\right)^{-1}\cdot\sqrt{\left(\sqrt{3}+3\right)^2}=$$

$$=\left(3-\sqrt{2}+\sqrt{3}+\sqrt{2}\right)^{-1}\cdot\left(\sqrt{3}+3\right)=\frac{1}{\sqrt{3}+3}\cdot\left(\sqrt{3}+3\right)=\mathbf{1}$$

2. $\sqrt{\left(1-\sqrt{3}\right)^2}+\sqrt[5]{-\sqrt[3]{-\sqrt{1}}}+\sqrt{\frac{1}{2}\sqrt{2\sqrt{2\sqrt{2}}}-\frac{1}{2}\sqrt[8]{32\sqrt{32}}}$

$$=\sqrt{3}-1+\sqrt[15]{1}+\sqrt{\frac{1}{2}\sqrt{2\sqrt[4]{2^3}}-\frac{1}{2}\sqrt[8]{2^5\sqrt{2^5}}}=\sqrt{3}-1+1+\sqrt{\frac{1}{2}\sqrt[8]{2^7}-\frac{1}{2}\sqrt[16]{2^{15}}}=\sqrt{3}+\sqrt[16]{\frac{1}{2}-\frac{1}{2}\sqrt[16]{2^{15}}}=$$

$$=\sqrt{3}+\sqrt[16]{\frac{1}{2}\cdot\frac{\sqrt[16]{2^{15}}}{\sqrt[16]{2^{15}}}-\frac{1}{3}\sqrt[16]{2^{15}}}=\sqrt{3}+\frac{1}{2}\sqrt[16]{2^{15}}-\frac{1}{2}\sqrt[16]{2^{15}}=\sqrt{3}$$

Risolvi:

3. $\frac{\sqrt{3}+x}{2+\sqrt{3}}=\frac{2\sqrt{2}-\sqrt{6}}{\sqrt{2}}$

$$\frac{\sqrt{3}+x}{2+\sqrt{3}}=\frac{\sqrt{2}\left(2-\sqrt{3}\right)}{\sqrt{2}} \qquad \sqrt{3}+x=\left(2-\sqrt{3}\right)\cdot\left(2+\sqrt{3}\right) \qquad x=\mathbf{1-\sqrt{3}}$$

4. $\frac{x}{3-\sqrt{6}}+\frac{1-x}{3+\sqrt{6}}+3>-\frac{\sqrt{6}}{3}$

$$\frac{x\left(3+\sqrt{6}\right)+(1-x)\left(3-\sqrt{6}\right)+9}{\left(3-\sqrt{6}\right)\left(3+\sqrt{6}\right)}>-\frac{\sqrt{6}}{3} \qquad 3x+x\sqrt{6}+3-\sqrt{6}-3x+x\sqrt{6}+9>-\sqrt{6}$$

$$2x\sqrt{6}>-12 \qquad x>-\frac{6}{\sqrt{6}} \qquad \boldsymbol{x>-\sqrt{6}}$$

5. $(\sqrt{5}-x)(\sqrt{5}+x)-(\sqrt{7}+x)(x-2\sqrt{7})+(2x+1)(x-3)<x\,(1+\sqrt{7})$

$$5-x^2-(x\sqrt{7}-14+x^2-2x\sqrt{7})+2x^2-6x+x-3<x+\;x\sqrt{7}$$

$$5-x^2+x\sqrt{7}+14-x^2+2x^2-6x+x-3<x+\;x\sqrt{7}$$

$$-6x<-16 \qquad \boldsymbol{x>\frac{8}{3}}$$

6. $\frac{3x+3\sqrt{3}}{\sqrt{3}\,(3x-\sqrt{3})}\geq 0$

$N\geq 0: \quad x\geq -\sqrt{3}$

$D>0: \quad x>\frac{\sqrt{3}}{3}$

$$\boldsymbol{x\leq -\sqrt{3}} \quad \vee \quad \boldsymbol{x>\frac{\sqrt{3}}{3}}$$

7. $(x+2)^2+1\leq 4\,(x+2)$

$$x^2+4x+4+1\leq 4x+8 \qquad x^2-3\leq 0 \qquad (x-\sqrt{3})(x+\sqrt{3})\leq 0$$

$$\boldsymbol{-\sqrt{3}\leq x\leq \sqrt{3}}$$

8. $\begin{cases} x\sqrt{3}+y\sqrt{2}=0 \\ x+y=\sqrt{3}-\sqrt{2} \end{cases}$

$$\begin{cases} x=-y\frac{\sqrt{2}}{\sqrt{3}} \\ -y\frac{\sqrt{2}}{\sqrt{3}}+y=\sqrt{3}-\sqrt{2} \end{cases} \qquad \begin{cases} x=-y\frac{\sqrt{2}}{\sqrt{3}} \\ y\left(-\frac{\sqrt{2}}{\sqrt{3}}+1\right)=\sqrt{3}-\sqrt{2} \end{cases} \qquad \begin{cases} x=-y\frac{\sqrt{2}}{\sqrt{3}} \\ y\frac{-\sqrt{2}+\sqrt{3}}{\sqrt{3}}=\sqrt{3}-\sqrt{2} \end{cases}$$

$$\begin{cases} x=-y\frac{\sqrt{2}}{\sqrt{3}} \\ \frac{y}{\sqrt{3}}=1 \end{cases} \qquad \begin{cases} x=-\sqrt{3}\frac{\sqrt{2}}{\sqrt{3}} \\ y=\sqrt{3} \end{cases} \qquad \begin{cases} \boldsymbol{x=-\sqrt{2}} \\ \boldsymbol{y=\sqrt{3}} \end{cases}$$

Semplifica la seguente espressione letterale:

9. $\sqrt[3]{\frac{1}{2-a}\sqrt{a-2}}$

$$C.E.: \; a-2>0 \quad \Longrightarrow \quad a>2$$

$$-\sqrt[3]{\frac{1}{a-2}\sqrt{a-2}} = -\sqrt[3]{\sqrt{\frac{a-2}{(a-2)^2}}} = -\sqrt[6]{\frac{\mathbf{1}}{\boldsymbol{a-2}}}$$

Considera le seguenti espressioni contenenti radicali:

A. Trasformale in espressioni con esponenti frazionari

B. Semplificale utilizzando le proprietà delle potenze

C. Riscrivi i risultati sotto forma di radicale

10. $\sqrt[3]{\frac{9\sqrt{2}}{4\sqrt{3}}} \cdot \sqrt{\frac{3\sqrt{2}}{2\sqrt{3}}}$

$$= \left(3^2 \cdot 2^{\frac{1}{2}} \cdot 2^{-2} \cdot 3^{-\frac{1}{2}}\right)^{\frac{1}{3}} \cdot \left(3 \cdot 2^{\frac{1}{2}} \cdot 2^{-1} \cdot 3^{-\frac{1}{2}}\right)^{\frac{1}{2}} = \left(3^{\frac{3}{2}} \cdot 2^{-\frac{3}{2}}\right)^{\frac{1}{3}} \cdot \left(3^{\frac{1}{2}} \cdot 2^{-\frac{1}{2}}\right)^{\frac{1}{2}} =$$

$$= 3^{\frac{1}{2}} \cdot 2^{-\frac{1}{2}} \cdot 3^{\frac{1}{4}} \cdot 2^{-\frac{1}{4}} = 3^{\frac{3}{4}} \cdot 2^{-\frac{3}{4}} = \left(\frac{3}{2}\right)^{\frac{3}{4}} = \sqrt[4]{\frac{\mathbf{27}}{\mathbf{8}}}$$

11. $\sqrt[3]{4\sqrt{2}} \cdot \sqrt{\frac{1}{2}\sqrt[3]{2}} \cdot \sqrt{\frac{1}{2}}$

$$= \left(2^2 \cdot 2^{\frac{1}{2}}\right)^{\frac{1}{3}} \cdot \left(2^{-1} \cdot 2^{\frac{1}{3}}\right)^{\frac{1}{2}} \cdot 2^{-\frac{1}{2}} = \left(2^{\frac{5}{2}}\right)^{\frac{1}{3}} \cdot \left(2^{-\frac{2}{3}}\right)^{\frac{1}{2}} \cdot 2^{-\frac{1}{2}} = 2^{\frac{5}{6}} \cdot 2^{-\frac{1}{3}} \cdot 2^{-\frac{1}{2}} =$$

$$= 2^{\frac{5}{6}-\frac{1}{3}-\frac{1}{2}} = 2^{\frac{5-2-3}{6}} = 2^0 = \mathbf{1}$$