Risolvi le seguenti equazioni:

1. $\left(x+\frac{1}{2}\right)^2-\left(\frac{1}{2}-x\right)\left(\frac{1}{2}+x\right)=\left(x-\frac{3}{2}\right)(x+2)+6$

$x^2+x+\frac{1}{4}-\frac{1}{4}+x^2=x^2+2x-\frac{3}{2}x-3+6$ $\quad 2x^2+x-6=0$ $\quad x_{1,2}=\frac{-1\pm 7}{4}=\begin{cases}\frac{3}{2}\\ -2\end{cases}$

2. $\frac{(x-\sqrt{3})^2}{\sqrt{2}}-\frac{(x+\sqrt{2})^2}{\sqrt{3}}=\frac{(x^2+3)\sqrt{3}-14\sqrt{2}}{\sqrt{6}}-2x\sqrt{6}$

$\sqrt{3}(x^2-2x\sqrt{3}+3)-\sqrt{2}(x^2+2x\sqrt{2}+2)-x^2\sqrt{3}-3\sqrt{3}+14\sqrt{2}+12x=0$

$x^2\sqrt{3}-6x+3\sqrt{3}-x^2\sqrt{2}-4x-2\sqrt{2}-x^2\sqrt{3}-3\sqrt{3}+14\sqrt{2}+12x=0$

$x^2\sqrt{2}-2x-12\sqrt{2}=0$ $\quad x^2-x\sqrt{2}-12=0$ $\quad x_{1,2}=\frac{\sqrt{2}\pm\sqrt{2+48}}{2}=\frac{\sqrt{2}\pm 5\sqrt{2}}{2}=\begin{cases}3\sqrt{2}\\ -2\sqrt{2}\end{cases}$

3. $\frac{x}{x^2-2}+\frac{1}{x^2-2x\sqrt{2}+2}=-\frac{1}{x+\sqrt{2}}$

$\frac{x}{(x-\sqrt{2})(x+\sqrt{2})}+\frac{1}{(x-\sqrt{2})^2}=-\frac{1}{x+\sqrt{2}}$ $\quad m.c.d.=(x+\sqrt{2})(x-\sqrt{2})^2$ $\quad C.A.: x\neq\pm\sqrt{2}$

$x(x-\sqrt{2})+x+\sqrt{2}+x^2-2x\sqrt{2}+2=0$ $\quad 2x^2+x(1-3\sqrt{2})+2+\sqrt{2}=0$

$\Delta=(1-3\sqrt{2})^2-8(2+\sqrt{2})=1+18-6\sqrt{2}-16-8\sqrt{2}=3-14\sqrt{2}<0$ $\quad \nexists x\in\mathbb{R}$

4. $\frac{1}{x^4-1}+\frac{2x+1}{x^3+x^2+x+1}=\frac{x}{x^3-x^2+x-1}$

$\frac{1}{(x^2+1)(x^2-1)}+\frac{2x+1}{x^2(x+1)+1(x+1)}-\frac{x}{x^2(x-1)+1(x-1)}=0$

$\frac{1}{(x^2+1)(x-1)(x+1)}+\frac{2x+1}{(x^2+1)(x+1)}-\frac{x}{(x^2+1)(x-1)}=0$

$m.c.d.=(x^2+1)(x-1)(x+1)$ $\quad C.A.: x\neq\pm 1$

$1+(2x+1)(x-1)-x(x+1)=0$ $\quad 1+2x^2-2x+x-1-x^2-x=0$ $\quad x^2-2x=0$ $\quad \begin{matrix}x_1=0\\ x_2=2\end{matrix}$

5. $\frac{1}{1+\frac{1}{x-1}}+\frac{1}{1-\frac{1}{x-1}}=-1$

$\frac{1}{\frac{x-1+1}{x-1}}+\frac{1}{\frac{x-1-1}{x-1}}=-1$ $\quad \frac{x-1}{x}+\frac{x-1}{x-2}=-1$ $\quad m.c.d.=x(x-2)$ $\quad C.A.: \begin{cases}x\neq 0\\ x\neq 1\\ x\neq 2\end{cases}$

$(x-1)(x-2)+x(x-1)+x(x-2)=0$ $\quad x^2-2x-x+2+x^2-x+x^2-2x=0$

$3x^2-6x+2=0$ $\quad x_{1,2}=\frac{3\pm\sqrt{3}}{3}$

Risolvi e discuti le seguenti equazioni letterali:

6. $kx^2 - 2(k-1)\,x = 0$

$Se\ \boldsymbol{k = 0}$ $\quad \boldsymbol{x = 0}$

$Se\ \boldsymbol{k \neq 0}$ $\quad x\,(kx - 2k + 2) = 0$ $\quad \boldsymbol{x_1 = 0}$, $\boldsymbol{x_2 = \dfrac{2k-2}{k}}$

7. $kx^2 - k^2 = 0$

$Se\ \boldsymbol{k = 0}$ $\quad \forall \boldsymbol{x} \in \mathbb{R}$

$Se\ k \neq 0$ $\quad x^2 = k$

$\boldsymbol{Se\ k > 0}$ $\quad \boldsymbol{x_{1,2} = \pm\sqrt{k}}$ $\qquad \boldsymbol{Se\ k < 0}$ $\quad \nexists \boldsymbol{x} \in \mathbb{R}$

8. $(k-1)\,x^2 - 2kx + k = 0$

$Se\ \boldsymbol{k = 1}$ $\quad \boldsymbol{x = \dfrac{1}{2}}$

$Se\ k \neq 1$ $\quad \dfrac{\Delta}{4} = k^2 - k\,(k-1) = k^2 - k^2 + k = k$

$Se\ \boldsymbol{k < 0}$ $\quad \nexists \boldsymbol{x} \in \mathbb{R}$

$Se\ \boldsymbol{k = 0}$ $\quad \boldsymbol{x_{1,2} = 0}$

$Se\ \boldsymbol{k > 0}\ \wedge\ \boldsymbol{k \neq 1}$ $\quad x_{1,2} = \boldsymbol{\dfrac{k \pm \sqrt{k}}{k-1}}$

Data l'equazione parametrica: $(k-1)\,x^2 - 2(k+2)\,x + k = 0$, stabilisci per quale valore di k:

le soluzioni sono reali e distinte: $\frac{\Delta}{4} > 0$: $(k+2)^2 - k(k-1) = k^2 + 4k + 4 - k^2 + k > 0$ $\quad \boldsymbol{k > -\frac{4}{5}}$

le soluzioni sono reali e opposte: $S = 0 \Rightarrow -\frac{b}{a} = 0 \Rightarrow b = 0 \Rightarrow k + 2 = 0 \Rightarrow k = -2 \Rightarrow \nexists \boldsymbol{k} \in \mathbb{R}$

le soluzioni sono reali e reciproche: $P = 1 \Rightarrow \frac{c}{a} = 1 \Rightarrow c = a \Rightarrow k = k - 1 \Rightarrow \nexists \boldsymbol{k} \in \mathbb{R}$

una delle soluzioni è nulla: $\boldsymbol{k = 0}$

una delle soluzioni è uguale a -4: $16k - 16 + 8k + 16 + k = 0 \Rightarrow 25k = 0 \Rightarrow \boldsymbol{k = 0}$