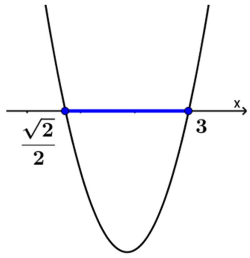
1. $2\,x^2-\left(6+\sqrt{2}\right)x+3\sqrt{2}\leq 0$

$$x_{1,2}=\frac{6+\sqrt{2}\pm\sqrt{36+2+12\sqrt{2}-24\sqrt{2}}}{4}=\frac{6+\sqrt{2}\pm\left(6-\sqrt{2}\right)}{4}=\left\langle \begin{array}{c}3\\ \frac{\sqrt{2}}{2}\end{array}\right.$$



$$\boldsymbol{\frac{\sqrt{2}}{2}\leq x\leq 3}$$

2. $-\left(x-\sqrt{3}\right)^2-\left(x+\sqrt{2}\right)^2-(x+1)^2>1$

$$\left(x-\sqrt{3}\right)^2+\left(x+\sqrt{2}\right)^2+(x+1)^2<-1 \qquad \boldsymbol{\nexists x\in\mathbb{R}}$$

Perché una somma di quadrati è sempre positiva

3. $\dfrac{(x^2+6)(x-4)^4}{8+x^2}>0$

$$x^2+6>0 \qquad \forall x\in\mathbb{R}$$
$$8+x^2>0 \qquad \forall x\in\mathbb{R}$$
$$(x-4)^4>0 \qquad \boldsymbol{\forall x\in\mathbb{R}-\{4\}}$$

4. $\dfrac{x^2-4x+4}{x^2-2x+1}\geq 1$

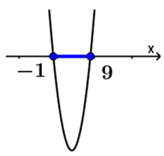
$$\frac{x^2-4x+4-x^2+2x-1}{x^2-2x+1}\geq 0 \qquad \frac{3-2x}{(x-1)^2}\geq 0$$

$$N\geq 0:\quad x\leq\frac{3}{2} \qquad D>0:\ \forall x\in\mathbb{R}-\{1\}$$

$$\boldsymbol{x\leq\frac{3}{2}\quad\wedge\quad x\neq 1}$$

5. $\begin{cases}16\,x^2-8x+1>0\\ x^2-8x-9\leq 0\end{cases}$

$$\begin{cases}(4x-1)^2>0\\ (x-9)(x+1)\leq 0\end{cases} \qquad \begin{cases}x\neq\dfrac{1}{4}\\ -1\leq x\leq 9\end{cases}$$

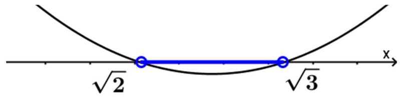


$$\boldsymbol{-1\leq x\leq 9\quad\wedge\quad x\neq\frac{1}{4}}$$

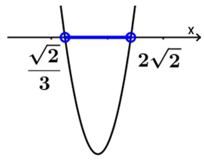
6. $\begin{cases} x^2 - x\sqrt{2} - x\sqrt{3} + \sqrt{6} < 0 \\ 3x^2 - 7x\sqrt{2} + 4 < 0 \\ 4x^2 + 4\sqrt{3}\,x + 3 > 0 \end{cases}$

$x\,(x-\sqrt{2}) - \sqrt{3}\,(x-\sqrt{2}) < 0 \qquad (x-\sqrt{2})(x-\sqrt{3}) < 0$

$\sqrt{2} < x < \sqrt{3}$

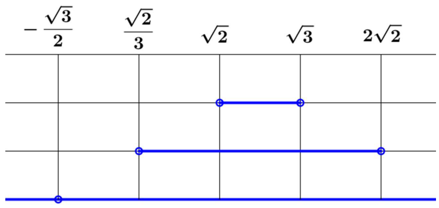


$x_{1,2} = \dfrac{7\sqrt{2} \pm \sqrt{98-48}}{6} = \left\langle \begin{array}{l} 2\sqrt{2} \\ \dfrac{\sqrt{2}}{3} \end{array} \right. \qquad \dfrac{\sqrt{2}}{3} < x < 2\sqrt{2}$



$(2x+\sqrt{3})^2 > 0 \qquad x \neq -\dfrac{\sqrt{3}}{2}$

$\begin{cases} \sqrt{2} < x < \sqrt{3} \\ \dfrac{\sqrt{2}}{3} < x < 2\sqrt{2} \\ x \neq -\dfrac{\sqrt{3}}{2} \end{cases}$



$\boldsymbol{\sqrt{2} < x < \sqrt{3}}$

7. $|\,x^2 - 8x + 19\,| \leq 3$

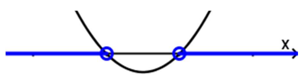
$-3 \leq x^2 - 8x + 19 \leq 3$

$\begin{cases} x^2 - 8x + 19 \geq -3 \\ x^2 - 8x + 19 \leq 3 \end{cases} \qquad \begin{cases} x^2 - 8x + 22 \geq 0 \\ x^2 - 8x + 16 \leq 0 \end{cases} \qquad \begin{cases} \forall x \in \mathbb{R} \\ (x-4)^2 \leq 0 \end{cases} \qquad \boldsymbol{x = 4}$

8. $ax^2 + (1-a^2)\,x - a \geq 0$

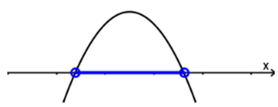
$\Delta = (1-a^2)^2 + 4a^2 = (1+a^2)^2 \geq 0 \qquad \forall a \in \mathbb{R}$

Se $\boldsymbol{a > 0}$: $\quad x_{1,2} = \dfrac{a^2 - 1 \pm (1+a^2)}{2a} = \left\langle \begin{array}{l} a \\ -\dfrac{1}{a} \end{array} \right.$



$\boldsymbol{x \leq -\dfrac{1}{a} \quad \vee \quad x \geq a}$

Se $\boldsymbol{a < 0}$: $\quad x_{1,2} = \dfrac{a^2 - 1 \pm (1+a^2)}{2a} = \left\langle \begin{array}{l} a \\ -\dfrac{1}{a} \end{array} \right.$



$\boldsymbol{a \leq x \leq -\dfrac{1}{a}}$

Se $\boldsymbol{a = 0}$: $\quad \boldsymbol{x \geq 0}$

9. Nell'equazione in x:

$$k^2 + k\,(x-2) = x\,(k+1) + 3$$

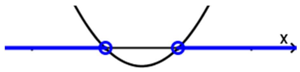
determina i valori di k affinché la radice sia positiva.

Determino innanzi tutto la soluzione dell'equazione:

$$k^2 + kx - 2k = xk + x + 3 \qquad\qquad x = k^2 - 2k - 3$$

Pongo la soluzione maggiore di zero:

$$k^2 - 2k - 3 > 0 \qquad\qquad k_{1,2} = \frac{1 \pm \sqrt{1+3}}{1} = \left\langle \begin{matrix} 3 \\ -1 \end{matrix} \right.$$



$$\boldsymbol{k < -1 \quad \vee \quad k > 3}$$