Risolvi le seguenti equazioni e disequazioni:

1. $25^x + (5^x)^2 - 5^{2(x-1)} = 245$

$$5^{2x} + 5^{2x} - \frac{5^{2x}}{25} = 245 \qquad 5^{2x}\left(1 + 1 - \frac{1}{25}\right) = 245$$

$$5^{2x} \cdot \frac{49}{25} = 245 \qquad 5^{2x-2} = 5 \qquad 2x - 2 = 1 \qquad x = \mathbf{\frac{3}{2}}$$

2. $\dfrac{\sqrt[3]{3^{x-2}} \cdot 27^{x+1}}{54\sqrt{9^{x-4}}} = \dfrac{243}{2}$

$$\frac{3^{\frac{x-2}{3}} \cdot 3^{3(x+1)}}{2 \cdot 3^3 \cdot 3^{x-4}} = \frac{3^5}{2} \qquad \frac{3^{\frac{x-2}{3}+3x+3}}{3^{3+x-4}} = 3^5$$

$$3^{\frac{x-2}{3}+3x+3-3-x+4} = 3^5 \qquad x - 2 + 9x - 3x + 12 = 15 \qquad x = \mathbf{\frac{5}{7}}$$

3. $3^{8x+3} - 81^{x+1} \geq 9^{2x} - 3$

$$3^{8x} \cdot 3^3 - 3^{4x} \cdot 81 \geq 3^{4x} - 3 \qquad \textit{Pongo } 3^{4x} = t$$

$$27\,t^2 - 81\,t \geq t - 3 \qquad 27\,t\,(t-3) - (t-3) \geq 0 \qquad (t-3)(27t-1) \geq 0$$

$$t \leq \frac{1}{27} \quad \vee \quad t \geq 3$$

$$3^{4x} \leq \frac{1}{27} \quad \vee \quad 3^{4x} \geq 3 \qquad 3^{4x} \leq 3^{-3} \quad \vee \quad 3^{4x} \geq 3^1$$

$$\boldsymbol{x \leq -\frac{3}{4}} \quad \vee \quad \boldsymbol{x \geq \frac{1}{4}}$$

4. $\log\,(3-x) + \frac{1}{2}\log\,(x+1) = \frac{1}{2}\log\,(3-x)$

$$CA: \begin{cases} 3 - x > 0 \\ x + 1 > 0 \end{cases} \qquad \begin{cases} x < 3 \\ x > -1 \end{cases} \qquad -1 < x < 3$$

$$2\,\log\,(3-x) + \log\,(x+1) = \log\,(3-x)$$

$$\log\,(3-x) + \log\,(x+1) = 0$$

$$\log\,(3-x)(x+1) = \log\,1 \qquad 3x + 3 - x^2 - x = 1$$

$$x^2 - 2x - 2 = 0 \qquad x_{1,2} = \mathbf{1 \pm \sqrt{3}}$$

soluzioni entrambe accettabili

5. $\log_6(x^2+18x+17) \le 2$

$$\log_6 (x^2+18x+17) \le \log_6 36$$

$$\begin{cases} x^2+18x+17>0 \\ x^2+18x+17 \le 36 \end{cases} \qquad \begin{cases} x^2+18x+17>0 \\ x^2+18x-19 \le 0 \end{cases}$$

$$x_{1,2} = \frac{-9 \pm \sqrt{81-17}}{1} \begin{cases} -1 \\ -17 \end{cases} \qquad x_{1,2} = \frac{-9 \pm \sqrt{81+19}}{1} \begin{cases} 1 \\ -19 \end{cases}$$

$$\begin{cases} x<-17 \quad \vee \quad x>-1 \\ -19 \le x \le 1 \end{cases} \qquad \mathbf{-19 \le x < -17 \quad \vee \quad -1 < x \le 1}$$

6. $3^x = 16 \cdot 3^{1-x} + 2$

$$3^x - 16 \cdot \frac{3}{3^x} - 2 = 0 \qquad Pongo\ 3^x = t$$

$$t - \frac{48}{t} - 2 = 0 \qquad t^2 - 2t - 48 = 0 \qquad t_{1,2} = \frac{1 \pm \sqrt{1+48}}{1} \begin{cases} 8 \\ -6 \quad non\ accettabile \end{cases}$$

$$3^x = 8 \qquad x = \mathbf{\log_3 8 = \frac{\ln 8}{\ln 3}}$$

7. $\dfrac{7^{x+2} \cdot 3^{1+x}}{5^{2-x}} \le 27$

$$\frac{7^x \cdot 49 \cdot 3 \cdot 3^x}{\frac{25}{5^x}} \le 27 \qquad 7^x \cdot 3^x \cdot 5^x \le \frac{27 \cdot 25}{49 \cdot 3} \qquad 105^x \le \frac{225}{49}$$

$$x \le \log_{105} \frac{225}{49} \qquad \mathbf{x \le 2\frac{\ln 5 + \ln 3 - \ln 7}{\ln 5 + \ln 3 + \ln 7}}$$

Determina il dominio delle seguenti funzioni:

8. $y = \log_2(3x-1) + \frac{\ln(3-x)}{3^x - 9}$

$$\begin{cases} 3x-1>0 \\ 3-x>0 \\ 3^x - 9 \ne 0 \end{cases} \qquad \begin{cases} x > \frac{1}{3} \\ x<3 \\ x \ne 2 \end{cases} \qquad \mathbf{\frac{1}{3} < x < 3 \quad \wedge \quad x \ne 2}$$

9. $y = \sqrt{3 - \log_2 (x+5)}$ Esame di Stato Liceo Scientifico – Sessione ordinaria 2014: Quesito 9

$\begin{cases} x+5>0 \\ 3-\log_2(x+5) \geq 0 \end{cases}$ $\begin{cases} x>-5 \\ \log_2 (x+5) \leq 3 \end{cases}$ $\begin{cases} x>-5 \\ x+5 \leq 8 \end{cases}$ $\begin{cases} x>-5 \\ x \leq 3 \end{cases}$ $\boldsymbol{-5 < x \leq 3}$

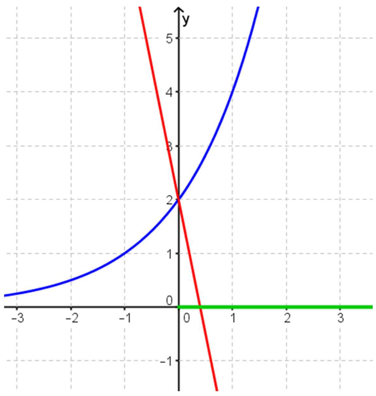
10. $y = \sqrt{\frac{\ln^2 x - \ln x}{3^x+2}} + 3^{\frac{5\sqrt{x-1}}{x^2+1}}$

$\begin{cases} \frac{\ln^2 x - \ln x}{3^x+2} \geq 0 \\ x>0 \\ x-1 \geq 0 \\ x^2+1 \neq 0 \end{cases}$ $\begin{cases} \ln^2 x - \ln x \geq 0 \\ x>0 \\ x \geq 1 \\ \forall x \in \mathbb{R} \end{cases}$ $\begin{cases} \ln x \leq 0 \quad \vee \quad \ln x \geq 1 \\ x>0 \\ x \geq 1 \end{cases}$ $\begin{cases} x \leq 1 \quad \vee \quad x \geq e \\ x>0 \\ x \geq 1 \end{cases}$

$\boldsymbol{x \geq e \quad \vee \quad x = 1}$

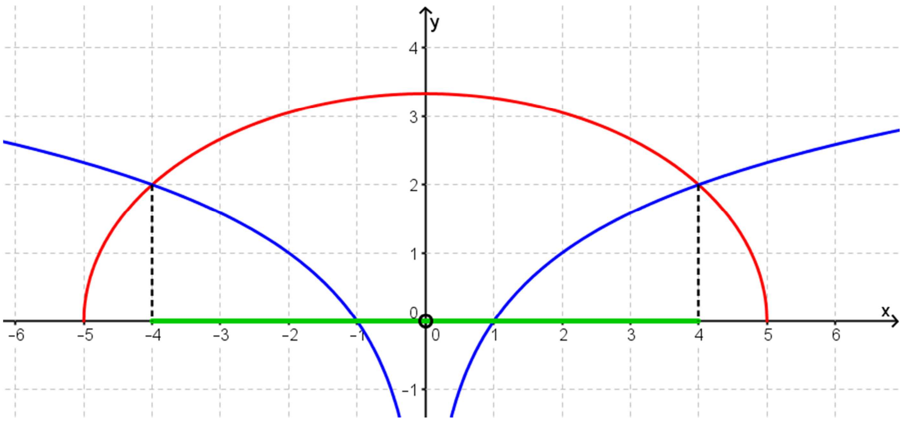
Risolvi graficamente le seguenti disequazioni:

11. $2^{x+1} \geq 2 - 5x$



$\boldsymbol{x \geq 0}$

12. $\log_2 |x| < \frac{\sqrt{100-4x^2}}{3}$



La funzione

$$y = \frac{\sqrt{100-4x^2}}{3}$$

è la metà superiore di un'ellisse con centro nell'origine degli assi cartesiani:

$$4x^2 + 9y^2 = 100$$

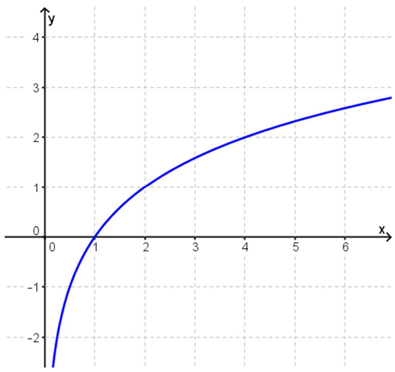
$$\frac{x^2}{25} + \frac{9y^2}{100} = 1$$

Che incontra la logaritmica nei punti di ascissa 4, perciò:

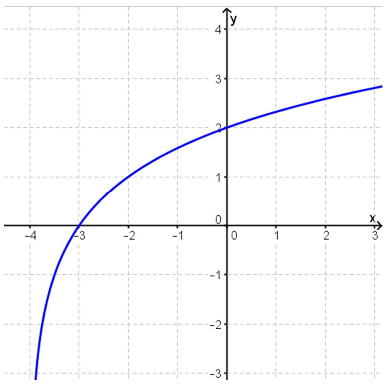
$\boldsymbol{-4 < x < 4 \quad \wedge \quad x \neq 0}$

Rappresenta le seguenti funzioni

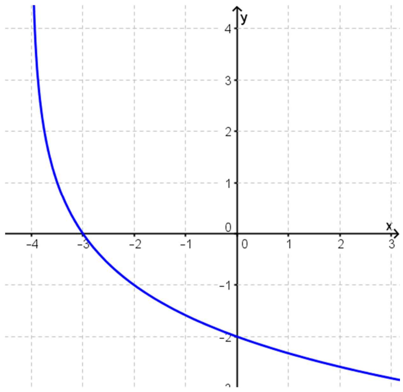
13. $y = -\log_2\ (x+4)$



$y = \log_2 x$

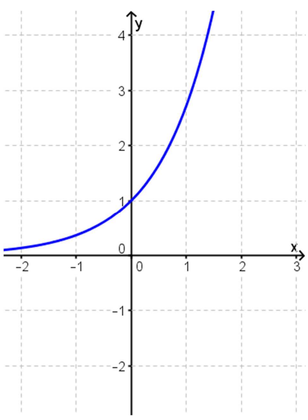


$y = \log_2(x+4)$

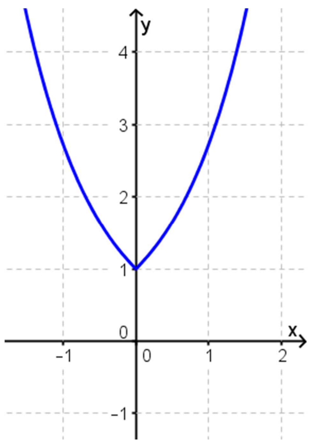


$y = -\log_2(x+4)$

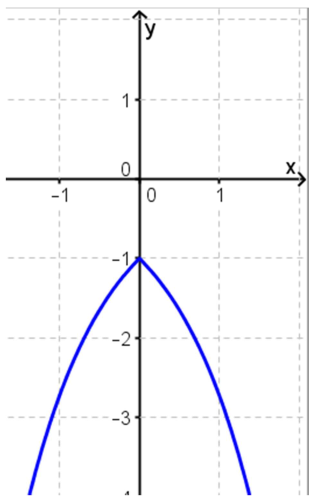
14. $y = -e^{|x|} + 1$



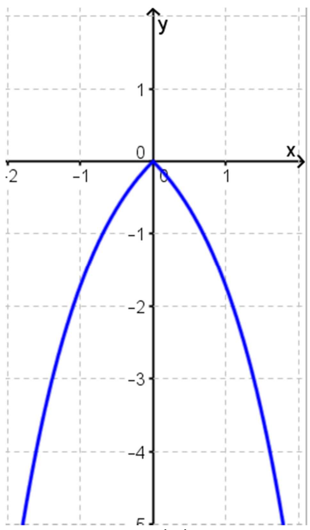
$y = e^x$



$y = e^{|x|}$



$y = -e^{|x|}$



$y = -e^{|x|} + 1$

15. Il valore dell'espressione $\log_2 3 \cdot \log_3 2$ è 1. Dire se questa affermazione è vera o falsa e fornire una esauriente spiegazione della risposta.

Esame di Stato Liceo Scientifico – Sessione ordinaria 2003: Quesito 10

Applicando la formula del cambiamento di base, possiamo dimostrare che l'affermazione è **VERA**:

$$\log_2 3 \cdot \log_3 2 = \log_2 3 \cdot \frac{\log_2 2}{\log_2 3} = \log_2 2 = \mathbf{1}$$