1. $\sqrt{\dfrac{\sqrt{12}-\sqrt{27}+\sqrt{75}}{\sqrt{6}-\sqrt{3}}+\dfrac{1}{1+\sqrt{2}}-3\sqrt{2}\cdot\dfrac{1}{\sqrt{\left(1-\sqrt{2}\right)^2}}}$

$$=\sqrt{\dfrac{2\sqrt{3}-3\sqrt{3}+5\sqrt{3}}{\sqrt{3}\left(\sqrt{2}-1\right)}+\dfrac{1}{1+\sqrt{2}}-3\sqrt{2}\cdot\dfrac{1}{\sqrt{2}-1}}=\sqrt{\dfrac{4\sqrt{3}}{\sqrt{3}\left(\sqrt{2}-1\right)}+\dfrac{1}{1+\sqrt{2}}-3\sqrt{2}\cdot\dfrac{1}{\sqrt{2}-1}}=$$

$$=\sqrt{\dfrac{4}{\sqrt{2}-1}+\dfrac{1}{1+\sqrt{2}}-3\sqrt{2}\cdot\dfrac{1}{\sqrt{2}-1}}=\sqrt{\dfrac{4+4\sqrt{2}+\sqrt{2}-1-3\sqrt{2}}{\left(\sqrt{2}-1\right)\left(1+\sqrt{2}\right)}\cdot\dfrac{1}{\sqrt{2}-1}}=$$

$$=\sqrt{3+2\sqrt{2}}\cdot\dfrac{1}{\sqrt{2}-1}=\sqrt{\left(\sqrt{2}+1\right)^2}\cdot\dfrac{1}{\sqrt{2}-1}=\dfrac{\sqrt{2}+1}{\sqrt{2}-1}\cdot\dfrac{\sqrt{2}+1}{\sqrt{2}+1}=\mathbf{3+2\sqrt{2}}$$

2. $\left(\sqrt{\dfrac{125}{27}}:\sqrt{\dfrac{5}{3}}+\dfrac{\sqrt{45}}{\sqrt{20}}+\dfrac{\sqrt{18}}{\sqrt{8}}\right):\sqrt{98}$

$$=\left(\sqrt{\dfrac{125}{27}\cdot\dfrac{3}{5}}+\dfrac{\sqrt{9}}{\sqrt{4}}+\dfrac{\sqrt{9}}{\sqrt{4}}\right):\left(7\sqrt{2}\right)=\left(\sqrt{\dfrac{25}{9}}+\dfrac{3}{2}+\dfrac{3}{2}\right):\left(7\sqrt{2}\right)=$$

$$=\left(\dfrac{5}{3}+3\right):\left(7\sqrt{2}\right)=\dfrac{14}{3}\cdot\dfrac{1}{7\sqrt{2}}=\dfrac{2}{3\sqrt{2}}\cdot\dfrac{\sqrt{2}}{\sqrt{2}}=\dfrac{\mathbf{\sqrt{2}}}{\mathbf{3}}$$

3. $\left(1+\sqrt{3}-\sqrt[4]{12}\right)\left(1+\sqrt{3}+\sqrt[4]{12}\right)-\sqrt{6\sqrt{3}-2\sqrt{23}}\cdot\sqrt{6\sqrt{3}+2\sqrt{23}}$

$$=\left(1+\sqrt{3}\right)^2-\sqrt{12}-\sqrt{108-92}=1+2\sqrt{3}+3-2\sqrt{3}-4=\mathbf{0}$$

4. $\dfrac{x}{\sqrt{2}}-\dfrac{x-3}{3\sqrt{2}}-\dfrac{x}{2}=0$

$$\dfrac{3x\sqrt{2}-x\sqrt{2}+3\sqrt{2}-3x}{6}=0 \qquad\qquad \left(2\sqrt{2}-3\right)x=-3\sqrt{2}$$

$$x=\dfrac{3\sqrt{2}}{3-2\sqrt{2}}\cdot\dfrac{3+2\sqrt{2}}{3+2\sqrt{2}}=9\sqrt{2}+12 \qquad\qquad \mathbf{x=9\sqrt{2}+12}$$

5. $\dfrac{1}{3\sqrt{3}+x} = \dfrac{6\sqrt{3}}{27-x^2} + \dfrac{1}{x-3\sqrt{3}}$

$$\dfrac{3\sqrt{3}-x-6\sqrt{3}+x+3\sqrt{3}}{(3\sqrt{3}+x)(3\sqrt{3}-x)} = 0 \qquad\qquad C.A.: x \neq \pm 3\sqrt{3}$$

$$0x = 0 \qquad\qquad \forall x \in \mathbb{R} - \{-3\sqrt{3};\ +3\sqrt{3}\ \}$$

6. $\dfrac{1-\sqrt{2}}{\sqrt{2}-x} = \dfrac{x^2+\sqrt{2}}{x^2-x\sqrt{2}} + \dfrac{1-x}{x}$

$$\dfrac{x-x\sqrt{2}+x^2+\sqrt{2}-\sqrt{2}+x\sqrt{2}+x-x^2}{x(\sqrt{2}-x)} = 0 \qquad\qquad C.A.: x \neq \sqrt{2} \quad \wedge \quad x \neq 0$$

$$2x = 0 \qquad x = 0 \quad non\ accettabile\ per\ le\ c.a.: \ \nexists x \in \mathbb{R}$$

7. $\begin{cases} x + y = \sqrt{5} + 3 \\ x - y = \sqrt{5} - 3 \end{cases}$

$$\begin{cases} x + y = \sqrt{5} + 3 \\ \underline{x - y = \sqrt{5} - 3} \\ \quad 2x \quad\ = 2\sqrt{5} \\ \qquad x = \sqrt{5} \end{cases} \qquad\qquad \begin{cases} x + y = \sqrt{5} + 3 \\ \underline{-x + y = -\sqrt{5} + 3} \\ \quad 2y \quad\ = 6 \\ \qquad y = 3 \end{cases}$$

$$\begin{cases} x = \sqrt{5} \\ y = 3 \end{cases}$$

8. $6 + x\sqrt{2} \leq \sqrt{6}\left(1 + \sqrt{6}\right)$

$$6 + x\sqrt{2} \leq \sqrt{6} + 6 \qquad x\sqrt{2} \leq \sqrt{6} \qquad x \leq \sqrt{3}$$

9. $\dfrac{x\sqrt{2}+1}{\sqrt{2}+1} \geq \dfrac{3\sqrt{2}+x}{3\sqrt{2}-1}$

$$(x\sqrt{2}+1)(3\sqrt{2}-1) \geq (3\sqrt{2}+x)(\sqrt{2}+1) \qquad\qquad 6x - x\sqrt{2} + 3\sqrt{2} - 1 \geq 6 + 3\sqrt{2} + x\sqrt{2} + x$$

$$(5 - 2\sqrt{2})x \geq 7 \qquad\qquad x \geq \dfrac{7}{5-2\sqrt{2}} \cdot \dfrac{5+2\sqrt{2}}{5+2\sqrt{2}} \qquad\qquad x \geq \dfrac{7\left(5+2\sqrt{2}\right)}{17}$$

10. $\dfrac{x\sqrt{2}}{1+x} < \dfrac{1}{1-\sqrt{2}}$

$$\frac{x\sqrt{2} - 2x - 1 - x}{1+x} > 0 \qquad\qquad \frac{x\left(\sqrt{2} - 3\right) - 1}{1+x} > 0$$

$N > 0$: $x < \dfrac{1}{\sqrt{2} - 3} \cdot \dfrac{\sqrt{2} + 3}{\sqrt{2} + 3} \qquad x < -\dfrac{\sqrt{2} + 3}{7}$

$D > 0$: $x > -1$

$$\mathbf{-1 < x < -\dfrac{\sqrt{2} + 3}{7}}$$