

1.  $\frac{(3x-1)^2}{3} + \frac{x+3}{6} > 3x(x-1) - \frac{2-7x}{4}$

$$\frac{4(3x-1)^2 + 2(x+3) - 36x(x-1) + 3(2-7x)}{12} > 0$$

$$4(9x^2 - 6x + 1) + 2x + 6 - 36x^2 + 36x + 6 - 21x > 0$$

$$36x^2 - 24x + 4 + 2x + 6 - 36x^2 + 36x + 6 - 21x > 0$$

$$-7x > -16 \quad x < \frac{16}{7}$$

2.  $\frac{1-2x}{3} + \frac{1}{2} < 2x + \frac{1}{3} \left(1 - \frac{x}{2}\right) + 8$

$$2(1-2x) + 3 < 12x + 2\left(1 - \frac{x}{2}\right) + 48$$

$$2 - 4x + 3 < 12x + 2 - x + 48$$

$$-15x < 45 \quad x > -3$$

3.  $\frac{4}{x-3} \geq 2 - \frac{5x-4}{x-3}$

$$\frac{4 - 2(x-3) + 5x - 4}{x-3} \geq 0 \quad \frac{4 - 2x + 6 + 5x - 4}{x-3} \geq 0$$

$$\frac{3x+6}{x-3} \geq 0 \quad N \geq 0: \quad 3x+6 \geq 0 \quad x \geq -2 \\ D > 0: \quad x-3 > 0 \quad x > 3$$

	-2	3	
N	-	+	+
D	-	-	+
+	-	+	

$$x \leq -2 \quad \vee \quad x > 3$$

4.  $\frac{4x}{1-x} > \frac{2}{x-1} - 2$

$$\frac{4x}{1-x} > -\frac{2}{1-x} - 2 \quad \frac{4x+2+2(1-x)}{1-x} > 0 \quad \frac{4x+2+2-2x}{1-x} > 0$$

$$\frac{2x+4}{1-x} > 0 \quad N > 0: \quad 2x+4 > 0 \quad x > -2 \\ D > 0: \quad 1-x > 0 \quad x < 1$$

	-2	1	
N	-	+	+
D	+	+	-
-	+	-	

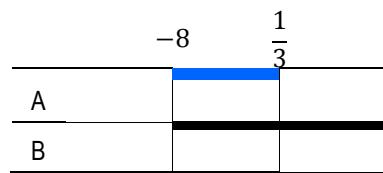
$$-2 < x < 1$$

5.  $\begin{cases} (x+3)(x-2) < (x+1)^2 + 1 \\ \frac{3x-1}{9} - \frac{6x-2}{6} + \frac{3x-1}{6} > x - \frac{1}{3} \end{cases}$

$$\begin{cases} x^2 - 2x + 3x - 6 < x^2 + 2x + 1 + 1 \\ 2(3x-1) - 3(6x-2) + 3(3x-1) > 18x - 6 \end{cases}$$

$$\begin{cases} -x < 8 \\ 6x - 2 - 18x + 6 + 9x - 3 > 18x - 6 \end{cases}$$

$$\begin{cases} x > -8 \\ -21x > -7 \end{cases} \quad \begin{cases} x > -8 \\ x < \frac{1}{3} \end{cases}$$



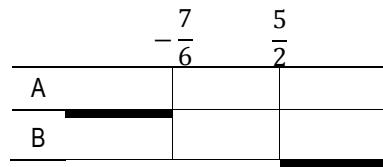
$$-8 < x < \frac{1}{3}$$

6.  $\begin{cases} (2x-1)^2 < 2(2x+1)(x-3) \\ (x-1)(x+1) > 2+x^2-2(x-1) \end{cases}$

$$\begin{cases} 4x^2 - 4x + 1 < 2(2x^2 - 6x + x - 3) \\ x^2 - 1 > 2 + x^2 - 2x + 2 \end{cases}$$

$$\begin{cases} 4x^2 - 4x + 1 < 4x^2 - 12x + 2x - 6 \\ 2x > 5 \end{cases}$$

$$\begin{cases} 6x < -7 \\ x > \frac{5}{2} \end{cases} \quad \begin{cases} x < -\frac{7}{6} \\ x > \frac{5}{2} \end{cases}$$



$$\nexists x \in \mathbb{R}$$

7.  $\frac{x^2 - 5x + 6}{x^2 - 1} \leq 0$

$$\frac{(x-2)(x-3)}{(x-1)(x+1)} \leq 0$$

$N_1 \geq 0:$	$x \geq 2$
$N_2 \geq 0:$	$x \geq 3$
$D_1 > 0:$	$x > 1$
$D_2 > 0:$	$x > -1$

	-1	1	2	3
$N_1$	-	-	-	+
$N_2$	-	-	-	+
$D_1$	-	-	+	+
$D_2$	-	+	+	+
	+	—	+	—

$$\textcolor{blue}{-1 < x < 1 \quad \vee \quad 2 \leq x \leq 3}$$

8.  $(x - 2a)(x + 2a) + 1 - x^2 > 3(1 - a^2) - 2ax$

$$x^2 - 4a^2 + 1 - x^2 > 3 - 3a^2 - 2ax$$

$$2ax > a^2 + 2$$

**Se  $a > 0$**      $x > \frac{a^2 + 2}{2a}$

**Se  $a = 0$**      $\nexists x \in \mathbb{R}$

**Se  $a < 0$**      $x < \frac{a^2 + 2}{2a}$

9.  $(x + a)^2 - b^2 \leq (x + b)(x - b) - a^2$

$$x^2 + 2ax + a^2 - b^2 \leq x^2 - b^2 - a^2$$

$$2ax \leq -2a^2$$

**Se  $a > 0$**      $x \leq -a$

**Se  $a = 0$**      $\forall x \in \mathbb{R}$

**Se  $a < 0$**      $x \geq -a$