1. $\sqrt{x^2+4} < x$

$$\begin{cases} x^2+4 \geq 0 \\ x \geq 0 \\ x^2+4 < x^2 \end{cases} \qquad \begin{cases} \forall x \in \mathbb{R} \\ x \geq 0 \\ \nexists x \in \mathbb{R} \end{cases} \qquad \boldsymbol{\nexists x \in \mathbb{R}}$$

2. $\sqrt[3]{-4x^3+1} - x > x+1$

$$\sqrt[3]{-4x^3+1} > 2x+1 \qquad -4x^3+1 > 8x^3+12x^2+6x+1$$

$$12x^3+12x^2+6x < 0 \qquad x\,(2x^2+2x+1) < 0$$

$$x_{1,2} = \frac{-1 \pm \sqrt{1-2}}{2} \quad \Rightarrow \quad 2x^2+2x+1 > 0 \quad \forall x \in \mathbb{R}$$

$$\boldsymbol{x < 0}$$

3. $\sqrt{6-2x+x^2} > x-1$

$$\begin{cases} x-1 \geq 0 \\ 6-2x+x^2 > x^2-2x+1 \end{cases} \quad \vee \quad \begin{cases} 6-2x+x^2 \geq 0 \\ x-1 < 0 \end{cases}$$

$$\begin{cases} x \geq 1 \\ \forall x \in \mathbb{R} \end{cases} \quad \vee \quad \begin{cases} \forall x \in \mathbb{R} \\ x < 1 \end{cases}$$

$$x \geq 1 \quad \vee \quad x < 1$$

$$\boldsymbol{\forall x \in \mathbb{R}}$$

4. $\sqrt{x^2-1} + x \leq |2x-3|$

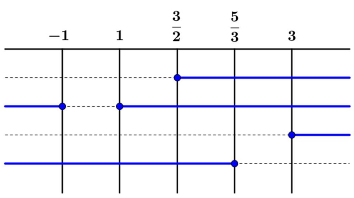
Vista la presenza del valore assoluto, ci sono due situazioni da considerare:

$$\begin{cases} 2x-3 \geq 0 \\ \sqrt{x^2-1}+x \leq 2x-3 \end{cases} \quad \vee \quad \begin{cases} 2x-3 < 0 \\ \sqrt{x^2-1}+x \leq -2x+3 \end{cases}$$

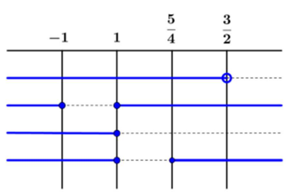
$$\begin{cases} 2x-3 \geq 0 \\ \sqrt{x^2-1} \leq x-3 \end{cases} \quad \vee \quad \begin{cases} 2x-3 < 0 \\ \sqrt{x^2-1} \leq -3x+3 \end{cases}$$

$$\begin{cases} 2x-3 \geq 0 \\ x^2-1 \geq 0 \\ x-3 \geq 0 \\ x^2-1 \leq x^2-6x+9 \end{cases} \quad \vee \quad \begin{cases} 2x-3 < 0 \\ x^2-1 \geq 0 \\ -3x+3 \geq 0 \\ x^2-1 \leq 9x^2+9-18\,x \end{cases}$$

$$\begin{cases} x \geq \frac{3}{2} \\ x \leq -1 \quad \vee \quad x \geq 1 \\ x \geq 3 \\ x \leq \frac{5}{3} \end{cases} \quad \vee \quad \begin{cases} x < \frac{3}{2} \\ x \leq -1 \quad \vee \quad x \geq 1 \\ x \leq 1 \\ 4x^2-9\,x+5 \geq 0 \qquad x \leq 1 \quad \vee \quad x \geq \frac{5}{4} \end{cases}$$



$\nexists x \in \mathbb{R}$



$\boldsymbol{x \leq -1 \quad \vee \quad x = 1}$

5. $\begin{cases} x^2+y^2-4x-6y-12=0 \\ x^2+y^2+12x-6y+20=0 \end{cases}$

$\begin{cases} x^2+y^2-4x-6y-12=0 \\ 16x+32=0 \end{cases}$ $\begin{cases} x=-2 \\ 4+y^2+8-6y-12=0 \end{cases}$ $\begin{cases} x=-2 \\ y^2-6y=0 \end{cases}$

$\begin{cases} \boldsymbol{x_1=-2} \\ \boldsymbol{y_1=0} \end{cases}$ $\begin{cases} \boldsymbol{x_2=-2} \\ \boldsymbol{y_2=6} \end{cases}$

6. $\begin{cases} x^2-xy=2\left(1-\sqrt{2}\right) \\ \left(\sqrt{2}+1\right)x=\left(1-\sqrt{2}\right)y \end{cases}$

$\begin{cases} y=\dfrac{\sqrt{2}+1}{1-\sqrt{2}}\,x=\dfrac{2+1+2\sqrt{2}}{-1}x=\left(-3-2\sqrt{2}\right)x \\ x^2-x^2\left(-3-2\sqrt{2}\right)=2\left(1-\sqrt{2}\right) \end{cases}$ $x^2\left(4+2\sqrt{2}\right)=2\left(1-\sqrt{2}\right)$ $x^2=\dfrac{1-\sqrt{2}}{2+\sqrt{2}}<0$

$\boldsymbol{\nexists x \in \mathbb{R}}$

7. Trova due numeri interi tali che il loro rapporto sia 2 e la differenza dei loro quadrati sia 147.

Indico con x e y i due numeri:

$\begin{cases} \dfrac{x}{y}=2 \\ x^2-y^2=147 \end{cases}$ $\begin{cases} x=2y \\ 4y^2-y^2=147 \end{cases}$ $\begin{cases} x=2y \\ y^2=49 \end{cases}$

$\begin{cases} \boldsymbol{x_1=14} \\ \boldsymbol{y_1=7} \end{cases}$ $\begin{cases} \boldsymbol{x_2=-14} \\ \boldsymbol{y_2=-7} \end{cases}$

8. Un rettangolo ha perimetro 124 cm. Calcola la sua area, sapendo che la diagonale misura 50 cm.

Indico le dimensioni del rettangolo con x e y. L'area sarà quindi data da xy:

$\begin{cases} 2\,(x+y)=124 \\ x^2+y^2=50^2 \end{cases}$ $\begin{cases} x+y=62 \\ (x+y)^2-2xy=50^2 \end{cases}$ $-2xy=50^2-62^2$ $xy=\dfrac{62^2-50^2}{2}$

Perciò il valore dell'area è $\boldsymbol{672\ cm^2}$