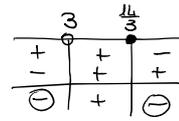


1. $\frac{3x-4}{x-3} \leq 6$

$$\frac{3x-4-6x+18}{x-3} \leq 0$$

$$\frac{-3x+14}{x-3} \leq 0$$

$$\begin{aligned} N \geq 0: & x \leq \frac{14}{3} \\ D > 0: & x > 3 \end{aligned}$$



$$x < 3 \vee x \geq \frac{14}{3}$$

2. $\left(1 - \frac{2}{x}\right)\left(3 + \frac{6}{x-2}\right) - \frac{x-3}{x-2} \leq 1$

$$\frac{x-2}{x} \cdot \frac{3x-6+6}{x-2} - \frac{x-3}{x-2} - 1 \leq 0$$

C.A.: $x \neq 0$

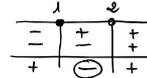
$$3 - \frac{x-3}{x-2} - 1 \leq 0$$

$$2 - \frac{x-3}{x-2} \leq 0$$

$$\frac{2x-4-x+3}{x-2} \leq 0$$

$$\frac{x-1}{x-2} \leq 0$$

$$\begin{aligned} N \geq 0: & x \geq 1 \\ D > 0: & x > 2 \end{aligned}$$



$$1 \leq x < 2$$

3. $x^4 + 2x^3 - x - 2 > 0$

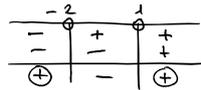
$$x^3(x+2) - (x+2) > 0$$

$$(x+2)(x^3 - 1) > 0$$

$$(x+2)(x-1)(x^2+x+1) > 0$$

Il terzo fattore è sicuramente positivo, in quanto si tratta di un falso quadrato:

$$\begin{aligned} IF > 0 & x > -2 \\ IIF > 0 & x > 1 \end{aligned}$$



$$x < -2 \vee x > 1$$

4. $(x+1)(x-2)^2 > (x+1)^2(x-2)$

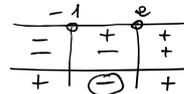
$$(x+1)(x-2)^2 - (x+1)^2(x-2) > 0$$

$$(x+1)(x-2)(x-2-x-1) > 0$$

$$-3(x+1)(x-2) > 0$$

$$(x+1)(x-2) < 0$$

$$\begin{aligned} IF > 0 & x > -1 \\ IIF > 0 & x > 2 \end{aligned}$$



$$-1 < x < 2$$

5. $\begin{cases} 4 + 3x^2 \geq 0 \\ \frac{1}{5}\left(\frac{1}{3} - 5x\right) < \frac{16}{15} \end{cases}$

$$4 + 3x^2 \geq 0 \quad \forall x \in \mathbb{R},$$

trattandosi di una somma di quadrati.

Perciò la soluzione del sistema è data dalla soluzione della seconda disequazione:

$$\frac{1}{15} - x < \frac{16}{15} \quad -x < 1 \quad x > -1$$

6. $\frac{6+x}{x-6} \leq 1$

$$\frac{6+x-x+6}{x-6} \leq 0$$

$$\frac{12}{x-6} \leq 0$$

$$x-6 < 0$$

$$x < 6$$

7. $\begin{cases} x(x+4) < 4(x-1) \\ x(x-4) \leq (x+6)(x-6) \end{cases}$

$$\begin{cases} x^2 + 4x < 4x - 4 \\ x^2 - 4x \leq x^2 - 36 \end{cases}$$

$$\begin{cases} x^2 < -4 \\ 4x \geq 36 \end{cases}$$

$$\begin{cases} \nexists x \in \mathbb{R} \\ x \geq 9 \end{cases}$$

$$\nexists x \in \mathbb{R}$$

$$8. (3x - 2a)^2 + (4x + a)^2 \leq (5x - a)^2 + 4a^2$$

$$9x^2 - 12ax + 4a^2 + 16x^2 + 8ax + a^2 \leq 25x^2 - 10ax + a^2 + 4a^2 \quad 6ax \leq 0$$

$$\text{Se } a = 0: \forall x \in \mathbb{R}$$

$$\text{Se } a > 0: x \leq 0$$

$$\text{Se } a < 0: x \geq 0$$

$$9. |2x - 3| = x + 1$$

$$\begin{cases} 2x - 3 \geq 0 \\ 2x - 3 = x + 1 \end{cases}$$

$$\begin{cases} x \geq \frac{3}{2} \\ x = 4 \end{cases}$$

$$\begin{cases} 2x - 3 < 0 \\ -2x + 3 = x + 1 \end{cases}$$

$$\begin{cases} x < \frac{3}{2} \\ x = \frac{2}{3} \end{cases}$$

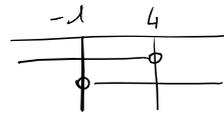
$$x = 4 \quad \vee \quad x = \frac{2}{3}$$

$$10. |2x - 3| < 5$$

$$\begin{cases} 2x - 3 < 5 \\ 2x - 3 > -5 \end{cases}$$

$$\begin{cases} 2x < 8 \\ 2x > -2 \end{cases}$$

$$\begin{cases} x < 4 \\ x > -1 \end{cases}$$



$$-1 < x < 4$$

$$11. \text{ Calcola, senza svolgere i calcoli: } \sqrt{999^2 + 999 + 1000}.$$

$$\sqrt{999^2 + 999 + 1000} = \sqrt{999(999 + 1) + 1000} = \sqrt{999 \cdot 1000 + 1000} = \sqrt{1000(999 + 1)} = \sqrt{1000^2} = 1000$$

$$12. \text{ Se il rapporto tra un numero e il suo precedente è minore di 1, come deve essere il numero?}$$

$$\frac{x}{x-1} < 1$$

$$\frac{x-x+1}{x-1} < 0$$

$$\frac{1}{x-1} < 0$$

$$x-1 < 0$$

$$x < 1$$