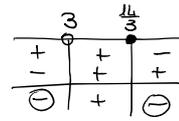


1.  $\frac{3x-4}{x-3} \leq 6$

$$\frac{3x-4-6x+18}{x-3} \leq 0$$

$$\frac{-3x+14}{x-3} \leq 0$$

$$\begin{aligned} N \geq 0: & x \leq \frac{14}{3} \\ D > 0: & x > 3 \end{aligned}$$



$$x < 3 \vee x \geq \frac{14}{3}$$

2.  $\left(1 + \frac{2}{x}\right)\left(3 - \frac{6}{x+2}\right) - \frac{x-1}{x+2} \leq 1$

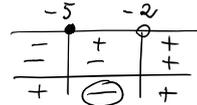
$$\frac{x+2}{x} \cdot \frac{3x+6-6}{x+2} - \frac{x-4}{x+2} - 1 \leq 0 \quad C.A.: x \neq 0$$

$$3 - \frac{x-4}{x+2} - 1 \leq 0$$

$$2 - \frac{x-1}{x+2} \leq 0$$

$$\frac{2x+4-x+1}{x+2} \leq 0$$

$$\frac{x+5}{x+2} \leq 0 \quad \begin{aligned} N \geq 0: & x \geq -5 \\ D > 0: & x > -2 \end{aligned}$$



$$-5 \leq x < -2$$

3.  $x^4 - 2x^3 - x + 2 > 0$

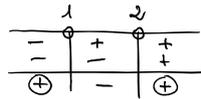
$$x^3(x-2) - (x-2) > 0$$

$$(x-2)(x^3-1) > 0$$

$$(x-2)(x-1)(x^2+x+1) > 0$$

Il terzo fattore è sicuramente positivo, in quanto si tratta di un falso quadrato:

$$\begin{aligned} IF > 0 & x > 2 \\ IIF > 0 & x > 1 \end{aligned}$$



$$x < 1 \vee x > 2$$

4.  $(x+2)(x-1)^2 > (x+2)^2(x-1)$

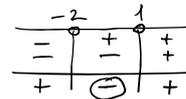
$$(x+2)(x-1)^2 - (x+2)^2(x-1) > 0$$

$$(x+2)(x-1)(x-1-x-2) > 0$$

$$-3(x+2)(x-1) > 0$$

$$(x+2)(x-1) < 0$$

$$\begin{aligned} IF > 0 & x > -2 \\ IIF > 0 & x > 1 \end{aligned}$$



$$-2 < x < 1$$

5.  $\begin{cases} 4 + 3x^2 \geq 0 \\ \frac{1}{5}\left(\frac{1}{3} - 5x\right) > \frac{16}{15} \end{cases}$

$4 + 3x^2 \geq 0 \quad \forall x \in \mathbb{R}$ , trattandosi di una somma di quadrati.

Perciò la soluzione del sistema è data dalla soluzione della seconda disequazione:

$$\frac{1}{15} - x > \frac{16}{15} \quad -x > 1 \quad x < -1$$

6.  $\frac{2+x}{x-2} \leq 1$

$$\frac{2+x-x+2}{x-2} \leq 0$$

$$\frac{4}{x-2} \leq 0$$

$$x-2 < 0$$

$$x < 2$$

7.  $\begin{cases} x(x+2) < 2(x-1) \\ x(x-8) \leq (x+4)(x-4) \end{cases}$

$$\begin{cases} x^2 + 2x < 2x - 2 \\ x^2 - 8x \leq x^2 - 16 \end{cases}$$

$$\begin{cases} x^2 < -2 \\ 8x \geq 16 \end{cases}$$

$$\begin{cases} \nexists x \in \mathbb{R} \\ x \geq 2 \end{cases}$$

$$\nexists x \in \mathbb{R}$$

$$8. (3x - 2a)^2 + (4x + a)^2 \geq (5x - a)^2 + 4a^2$$

$$9x^2 - 12ax + 4a^2 + 16x^2 + 8ax + a^2 \geq 25x^2 - 10ax + a^2 + 4a^2 \quad 6ax \geq 0$$

$$\text{Se } a = 0: \forall x \in \mathbb{R}$$

$$\text{Se } a > 0: x \geq 0$$

$$\text{Se } a < 0: x \leq 0$$

$$9. |3x - 2| = 2x + 1$$

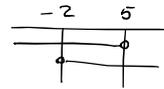
$$\begin{cases} 3x - 2 \geq 0 \\ 3x - 2 = 2x + 1 \end{cases} \quad \begin{cases} x \geq \frac{2}{3} \\ x = 3 \end{cases}$$

$$\begin{cases} 3x - 2 < 0 \\ -3x + 2 = 2x + 1 \end{cases} \quad \begin{cases} x < \frac{2}{3} \\ x = \frac{1}{5} \end{cases}$$

$$x = 3 \quad \vee \quad x = \frac{1}{5}$$

$$10. |2x - 3| < 7$$

$$\begin{cases} 2x - 3 < 7 \\ 2x - 3 > -7 \end{cases} \quad \begin{cases} 2x < 10 \\ 2x > -4 \end{cases} \quad \begin{cases} x < 5 \\ x > -2 \end{cases}$$



$$-2 < x < 5$$

$$11. \text{ Calcola, senza svolgere i calcoli: } \sqrt{999^2 + 999 + 1000}.$$

$$\sqrt{999^2 + 999 + 1000} = \sqrt{999(999 + 1) + 1000} = \sqrt{999 \cdot 1000 + 1000} = \sqrt{1000(999 + 1)} = \sqrt{1000^2} = 1000$$

$$12. \text{ Se il rapporto tra un numero e il suo precedente è maggiore di 1, come deve essere il numero?}$$

$$\frac{x}{x-1} > 1 \quad \frac{x-x+1}{x-1} > 0 \quad \frac{1}{x-1} > 0 \quad x-1 > 0 \quad x > 1$$