

Equazioni esponenziali

$$24. \quad (2^{x+3})^{x-4} = 1$$

$$(2^{x+3})^{x-4} = 2^0 \Rightarrow (x+3)(x-4) = 0 \Rightarrow x = -3; x = 4$$

$$25. \quad (2e^x + 3)(e^{2x-5} - 1) = 0$$

$$2e^x + 3 = 0 \Rightarrow e^x = -\frac{3}{2} \quad \text{impossibile}$$

$$e^{2x-5} - 1 = 0 \Rightarrow e^{2x-5} = e^0 \Rightarrow 2x - 5 = 0 \Rightarrow x = \frac{5}{2}$$

$$26. \quad \left(\frac{1}{5}\right)^{|x+4|} = 5^{2x}$$

$$5^{-|x+4|} = 5^{2x} \Rightarrow -|x+4| = 2x \Rightarrow$$

$$\begin{cases} x \geq -4 \\ -x - 4 = 2x \end{cases} \Rightarrow \begin{cases} x \geq -4 \\ x = -\frac{4}{3} \end{cases} \Rightarrow x = -\frac{4}{3}$$

$$\begin{cases} x < -4 \\ x + 4 = 2x \end{cases} \Rightarrow \begin{cases} x < -4 \\ x = 4 \end{cases} \quad \text{imp.}$$

$$27. \quad |2^{x+1} - 1| = 7$$

$$2^{x+1} - 1 = 7 \Rightarrow 2^{x+1} = 8 \Rightarrow 2^{x+1} = 2^3 \Rightarrow x + 1 = 3 \Rightarrow x = 2$$

$$2^{x+1} - 1 = -7 \Rightarrow 2^{x+1} = -6 \Rightarrow \text{imp.}$$

$$28. \quad \frac{5 \cdot 4^{x-2}}{4^x - 11} = 1$$

$$\text{Pongo: } 4^x = t \Rightarrow \frac{5 \cdot t \cdot 4^{-2}}{t - 11} = 1 \Rightarrow \frac{5}{16}t = t - 11 \quad \text{c.a.: } t \neq 11$$

$$-\frac{11}{16}t = -11 \Rightarrow t = 16 \Rightarrow 4^x = 16 \Rightarrow 4^x = 4^2 \Rightarrow x = 2$$

$$29. \quad 4^{\sqrt{|x+2|}} = 16$$

$$4^{\sqrt{|x+2|}} = 4^2 \Rightarrow \sqrt{|x+2|} = 2 \Rightarrow |x+2| = 4$$

$$x + 2 = 4 \Rightarrow x = 2$$

$$x + 2 = -4 \Rightarrow x = -6$$

30. $2^{2-x} - 2^{3-x} + 2^x = 0$

$$\frac{2^2}{2^x} - \frac{2^3}{2^x} + 2^x = 0$$

Pongo: $2^x = t \quad \frac{4}{t} - \frac{8}{t} + t = 0 \Rightarrow t^2 - 4 = 0 \Rightarrow t = \pm 2$

$t = 2 \Rightarrow 2^x = 2^1 \Rightarrow x = 1$

$t = -2 \Rightarrow 2^x = -2 \text{ imp.}$

31. $2^{2x+1} - 17 \cdot 2^x + 8 = 0$

$$(2^x)^2 \cdot 2^1 - 17 \cdot 2^x + 8 = 0$$

Pongo: $2^x = t \quad 2t^2 - 17t + 8 = 0$

$$t_{1,2} = \frac{17 \pm \sqrt{289 - 64}}{4} = \begin{cases} 8 \\ \frac{1}{2} \end{cases}$$

$t = 8 \Rightarrow 2^x = 8 \Rightarrow 2^x = 2^3 \Rightarrow x = 3$

$t = \frac{1}{2} \Rightarrow t = 2^{-1} \Rightarrow 2^x = 2^{-1} \Rightarrow x = -1$

32. $27 \frac{5x^2-3}{x^2+1} = 27 \frac{5-x^2}{3x^2+1}$

$$\frac{5x^2-3}{x^2+1} = \frac{5-x^2}{3x^2+1} \Rightarrow 15x^4 + 5x^2 - 9x^2 - 3 = 5x^2 + 5 - x^4 - x^2$$

$16x^4 - 8x^2 - 8 = 0 \Rightarrow 2x^4 - x^2 - 1 = 0 \quad \text{pongo } x^2 = t$

$$2t^2 - t - 1 = 0 \Rightarrow t_{1,2} = \frac{1 \pm \sqrt{1+8}}{4} = \begin{cases} 1 \\ -\frac{1}{2} \end{cases} \text{ non acc.}$$

$x^2 = 1 \Rightarrow x = \pm 1$

33. $3^{x+1} + 3^{x-2} - \frac{247}{3^{x-2}} = \frac{15}{3^{x-1}}$

$$3^x \cdot 3^1 + 3^x \cdot 3^{-2} - \frac{247}{3^x \cdot 3^{-2}} = \frac{15}{3^x \cdot 3^{-1}}$$

$3 \cdot 3^x + \frac{3^x}{9} - \frac{247 \cdot 9}{3^x} = \frac{15 \cdot 3}{3^x} \quad \text{pongo } 3^x = t$

$3t + \frac{t}{9} - \frac{2223}{t} - \frac{45}{t} = 0 \Rightarrow 27t^2 + t^2 - 9 \cdot (2223 + 45) = 0$

$28t^2 = 9 \cdot 2268 \Rightarrow t^2 = 9 \cdot 81 \Rightarrow t^2 = 3^2 \cdot 3^4 \Rightarrow$

$$t^2 = 3^6 \Rightarrow t = 3^3 \Rightarrow 3^x = 3^3 \Rightarrow x = 3$$

$$\Rightarrow t = -3^3 \text{ imp.}$$

$$34. \quad 13 \cdot 3^{1+x} - 3^{3-x} - \frac{13}{3^x} + 1 = 0$$

$$13 \cdot 3 \cdot 3^x - \frac{3^3}{3^x} - \frac{13}{3^x} + 1 = 0 \quad \text{pongo } 3^x = t$$

$$39t - \frac{27}{t} - \frac{13}{t} + 1 = 0 \Rightarrow 39t^2 + t - 40 = 0$$

$$\Rightarrow t_{1,2} = \frac{-1 \pm \sqrt{1 + 6240}}{78} = \begin{cases} \frac{-1 + 79}{78} = 1 \\ \frac{-1 - 79}{78} = -\frac{40}{39} \end{cases}$$

$$t = 1 \Rightarrow 3^x = 3^0 \Rightarrow x = 0$$

$$t = -\frac{40}{39} \Rightarrow 3^x = -\frac{40}{39} \Rightarrow \text{imp.}$$

$$35. \quad 4^{2-x} - 5 \cdot 2^{1-x} + 1 = 0$$

$$\frac{4^2}{4^x} - 5 \cdot \frac{2^1}{2^x} + 1 = 0 \quad \text{pongo } 2^x = t$$

$$\frac{16}{t^2} - \frac{10}{t} + 1 = 0 \Rightarrow$$

$$t^2 - 10t + 16 = 0 \Rightarrow t_{1,2} = \frac{5 \pm \sqrt{25 - 16}}{1} = \begin{cases} 8 \\ 2 \end{cases}$$

$$t = 8 \Rightarrow 2^x = 2^3 \Rightarrow x = 3$$

$$t = 2 \Rightarrow 2^x = 2^1 \Rightarrow x = 1$$

$$36. \quad \frac{3 \cdot 3^x + 3^{2-x} - 4}{3^x} = \frac{8}{3}$$

$$\frac{3 \cdot 3^x + \frac{3^2}{3^x} - 4}{3^x} = \frac{8}{3} \quad \text{pongo } 3^x = t$$

$$\frac{3t + \frac{3^2}{t} - 4}{t} = \frac{8}{3} \Rightarrow 9t^2 + 27 - 12t - 8t^2 = 0$$

$$t^2 - 12t + 27 = 0 \Rightarrow t_{1,2} = \frac{6 \pm \sqrt{36 - 27}}{1} = \begin{cases} 9 \\ 3 \end{cases}$$

$$t = 9 \Rightarrow 3^x = 3^2 \Rightarrow x = 2$$

$$t = 3 \Rightarrow 3^x = 3^1 \Rightarrow x = 1$$