37. $2^{5x+2} + 2^{5x+8} - 32^{x+1} = 912$

$2^{5x} \cdot 2^2 + 2^{5x} \cdot 2^8 - 32^x \cdot 32 = 912 \qquad pongo \;\; 2^{5x} = t$

$4\,t + 256\,t - 32\,t = 912 \quad \Rightarrow \quad 228\,t = 912$

$t = 4 \quad \Rightarrow \quad 2^{5x} = 2^2 \quad \Rightarrow \; 5x = 2 \quad \Rightarrow \quad \boxed{x = \frac{2}{5}}$

38. $5^{1+\sqrt{x}} + 5^{1-\sqrt{x}} = 10$

$5 \cdot 5^{\sqrt{x}} + \frac{5}{5^{\sqrt{x}}} = 10 \qquad pongo \;\; 5^{\sqrt{x}} = t$

$5\,t + \frac{5}{t} = 10 \quad \Rightarrow \quad 5t^2 - 10t + 5 = 0 \quad \Rightarrow \quad t^2 - 2t + 1 = 0$

$\Rightarrow \; (t-1)^2 = 0 \quad \Rightarrow \quad t = 1 \quad \Rightarrow \quad 5^{\sqrt{x}} = 5^0 \quad \Rightarrow \quad \boxed{x = 0}$

39. $\frac{5^{4x-1}}{5^{x-1}} - \frac{5^{3x}}{5^{x-1}} - \frac{5^{1-x}}{5^{1-2x}} + 5 = 0$

$5^{3x} - 5^{2x+1} - 5^x + 5 = 0 \qquad pongo \;\; 5^x = t$

$t^3 - 5t^2 - t + 5 = 0 \quad \Rightarrow \quad t^2\,(t-5) - 1\,(t-5) = 0$

$(t-5)\,(t^2-1) = 0 \quad \Rightarrow \quad (t-5)\,(t-1)\,(t+1) = 0$

$t = 5 \quad \Rightarrow \quad 5^x = 5^1 \quad \Rightarrow \quad \boxed{x = 1}$

$t = 1 \quad \Rightarrow \quad 5^x = 5^0 \quad \Rightarrow \quad \boxed{x = 0}$

$t = -1 \quad \Rightarrow \quad 5^x = -1 \quad \Rightarrow \quad imp.$

40. $50\left(\frac{4}{25}\right)^x - 133\left(\frac{2}{5}\right)^x + 20 = 0$

$50\left(\frac{2}{5}\right)^{2x} - 133\left(\frac{2}{5}\right)^x + 20 = 0 \qquad pongo \;\; \left(\frac{2}{5}\right)^x = t$

$50\,t^2 - 133\,t + 20 = 0 \quad \Rightarrow \quad t_{1,2} = \frac{133 \pm 117}{100} = \begin{cases} \frac{4}{25} \\ \frac{5}{2} \end{cases}$

$t = \frac{4}{25} \quad \Rightarrow \quad \left(\frac{2}{5}\right)^x = \left(\frac{2}{5}\right)^2 \quad \Rightarrow \quad \boxed{x = 2}$

$t = \frac{5}{2} \quad \Rightarrow \quad \left(\frac{2}{5}\right)^x = \left(\frac{5}{2}\right)^{-1} \quad \Rightarrow \quad \boxed{x = -1}$

41. $4\left(\frac{3}{2}\right)^{2x} + 15\left(\frac{3}{2}\right)^{-x} = 19$

$4\left(\left(\frac{3}{2}\right)^{x}\right)^{2} + 15\,\frac{1}{\left(\frac{3}{2}\right)^{x}} = 19 \qquad pongo\ \left(\frac{3}{2}\right)^{x} = t$

$4\,t^{2} + \frac{15}{t} = 19 \quad \Rightarrow \quad 4\,t^{3} - 19t + 15 = 0$

Applicando la regola di Ruffini: $(t-1)\,(4t^{2} + 4t - 15) = 0$

$t = 1 \quad \Rightarrow \quad \left(\frac{3}{2}\right)^{x} = \left(\frac{3}{2}\right)^{0} \quad \Rightarrow \quad x = 0$

$4t^{2} + 4t - 15 = 0 \quad \Rightarrow \quad t_{1,2} = \frac{-2 \pm \sqrt{4 + 60}}{4} = \begin{cases} \frac{3}{2} \\ -\frac{5}{2} \end{cases}$

$t = \frac{3}{2} \quad \Rightarrow \quad \left(\frac{3}{2}\right)^{x} = \left(\frac{3}{2}\right)^{1} \quad \Rightarrow \quad x = 1$

$t = -\frac{5}{2} \quad \Rightarrow \quad \left(\frac{3}{2}\right)^{x} = -\frac{5}{2} \quad \Rightarrow \quad imp.$

42. $5 \cdot 3^{x-1} - 3\,\frac{5^{2x-1}}{5^{x}} = 0$

$5 \cdot 3^{x-1} = 3 \cdot 5^{x-1} \quad \Rightarrow \quad \frac{5 \cdot 3^{x-1}}{3 \cdot 5^{x-1}} = \frac{3 \cdot 5^{x-1}}{3 \cdot 5^{x-1}} \quad \Rightarrow \quad \frac{5}{3} \cdot \left(\frac{3}{5}\right)^{x-1} = 1$

$\Rightarrow \quad \left(\frac{3}{5}\right)^{x-2} = \left(\frac{3}{5}\right)^{0} \quad \Rightarrow \quad x - 2 = 0 \quad \Rightarrow \quad x = 2$

43. $3 \cdot 3^{2x} + 7^{2x+1} = 3^{2x+2} + 7^{2x}$

$3 \cdot 3^{2x} - 3^{2} \cdot 3^{2x} + 7^{2x} \cdot 7^{1} - 7^{2x} = 0$

$3^{2x}\,(3 - 9) + 7^{2x}\,(7 - 1) = 0$

$-6 \cdot 3^{2x} + 6 \cdot 7^{2x} = 0 \quad \Rightarrow \quad 7^{2x} = 3^{2x} \quad \Rightarrow \quad \frac{7^{2x}}{3^{2x}} = \frac{3^{2x}}{3^{2x}} \quad \Rightarrow$

$\left(\frac{7}{3}\right)^{2x} = 1 \quad \Rightarrow \quad \left(\frac{7}{3}\right)^{2x} = \left(\frac{7}{3}\right)^{0} \quad \Rightarrow \quad 2x = 0 \quad \Rightarrow \quad x = 0$

44. $e^{2x} + e^{x} - 2 = 0$

Pongo: $e^{x} = t \quad \Rightarrow \quad t^2 + t - 2 = 0$

$$t_{1,2} = \frac{-1 \pm \sqrt{1+8}}{2} = \begin{cases} 1 \\ -2 \end{cases}$$

$t = 1 \quad \Rightarrow \quad e^{x} = e^{0} \quad \Rightarrow \quad x = 0$

$t = -2 \quad \Rightarrow \quad e^{x} = -2 \quad \Rightarrow \quad imp.$

45. $e^{2x} + e^{\frac{7}{3}} = e^{2+x} + e^{\frac{3x+1}{3}}$

$(e^{x})^2 + e^{\frac{7}{3}} - e^{2} \cdot e^{x} - e^{x} \cdot e^{\frac{1}{3}} = 0 \qquad pongo \quad e^{x} = t$

$t^2 + e^{\frac{7}{3}} - e^{2}\, t - e^{\frac{1}{3}} t = 0 \quad \Rightarrow \quad t^2 - t\,(e^{2} + e^{\frac{1}{3}}) + e^{\frac{7}{3}} = 0$

$$t_{1,2} = \frac{e^{2} + e^{\frac{1}{3}} \pm \sqrt{e^{4} + e^{\frac{2}{3}} + 2\, e^{\frac{7}{3}} - e^{\frac{7}{3}}}}{2} = \frac{e^{2} + e^{\frac{1}{3}} \pm (e^{2} - e^{\frac{1}{3}})}{2} = \begin{cases} e^{2} \\ e^{\frac{1}{3}} \end{cases}$$

$t = e^{2} \quad \Rightarrow \quad e^{x} = e^{2} \quad \Rightarrow \quad x = 2$

$t = \frac{1}{3} \quad \Rightarrow \quad e^{x} = e^{\frac{1}{3}} \quad \Rightarrow \quad x = \frac{1}{3}$

46. $e^{2x} + 3\, e^{x} - 4 = 0$

Pongo: $e^{x} = t \quad \Rightarrow \quad t^2 + 3\, t - 4 = 0$

$$t_{1,2} = \frac{-3 \pm \sqrt{9+16}}{2} = \begin{cases} 1 \\ -4 \end{cases}$$

$t = 1 \quad \Rightarrow \quad e^{x} = e^{0} \quad \Rightarrow \quad x = 0$

$t = -4 \quad \Rightarrow \quad e^{x} = -4 \quad \Rightarrow \quad imp.$

47. $e^{\frac{3x+8}{2}} - e^{\frac{x+2}{2}} + e^{x+3} - 1 = 0$

$e^{\frac{3}{2}x+4} - e^{\frac{1}{2}x+1} + e^{x+3} - 1 = 0$

$e^{\frac{3}{2}x} \cdot e^{4} - e^{\frac{1}{2}x} \cdot e + e^{x} \cdot e^{3} - 1 = 0 \qquad pongo \quad e^{\frac{1}{2}x} = t$

$e^{4}\, t^3 - e\, t + e^{3}\, t^2 - 1 = 0 \quad \Rightarrow \quad e^{3}\, t^2\,(et + 1) - (et + 1) = 0$

$\Rightarrow \quad (et + 1)\,(e^{3}\, t^2 - 1) = 0$

$et + 1 = 0 \quad \Rightarrow \quad t = -\frac{1}{e} \quad \Rightarrow \quad e^{\frac{1}{2}x} = -e^{-1} \quad \Rightarrow \quad imp.$

$e^{3}\, t^2 - 1 = 0 \quad \Rightarrow \quad t^2 = \frac{1}{e^{3}} \quad \Rightarrow \quad e^{x} = e^{-3} \quad \Rightarrow \quad x = -3$