

Disequazioni esponenziali

$$25. \quad 2^{\frac{x^2-x}{x+1}} \leq 1$$

$$2^{\frac{x^2-x}{x+1}} \leq 2^0 \quad \Rightarrow \quad \frac{x^2-x}{x+1} \leq 0$$

$$N \geq 0: x^2 - x \geq 0 \quad \Rightarrow \quad x \leq 0 \vee x \geq 1$$

$$D > 0: x + 1 > 0 \quad \Rightarrow \quad x > -1$$

$$x < -1 \vee 0 \leq x \leq 1$$

$$26. \quad \frac{2^x - 1}{8 - 2^x} \leq 0$$

$$N \geq 0: 2^x - 1 \geq 0 \quad \Rightarrow \quad x \geq 0$$

$$D > 0: 8 - 2^x > 0 \quad \Rightarrow \quad x < 3$$

$$x \leq 0 \vee x > 3$$

$$27. \quad 3^{\sqrt{3+x-2x^2}} < 3^{2-x}$$

$$\sqrt{3+x-2x^2} < 2-x$$

$$\begin{cases} 3+x-2x^2 \geq 0 \\ 2-x > 0 \\ 3+x-2x^2 < 4-4x+x^2 \end{cases} \quad \begin{cases} 2x^2-x-3 \leq 0 \\ x < 2 \\ 3x^2-5x+1 > 0 \end{cases}$$

$$\begin{cases} -1 \leq x \leq \frac{3}{2} \\ x < 2 \\ x < \frac{5-\sqrt{13}}{6} \vee x > \frac{5+\sqrt{13}}{6} \end{cases}$$

$$-1 \leq x < \frac{5-\sqrt{13}}{6} \vee \frac{5+\sqrt{13}}{6} < x \leq \frac{3}{2}$$

$$28. \quad \frac{5^{x^2-4x} - 1}{x-2} \geq 0$$

$$N \geq 0: 5^{x^2-4x} - 1 \geq 0 \quad \Rightarrow \quad x^2 - 4x \geq 0 \quad \Rightarrow \quad x \leq 0 \vee x \geq 4$$

$$D > 0: x - 2 > 0 \quad \Rightarrow \quad x > 2$$

$$0 \leq x < 2 \vee x \geq 4$$

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$$29. \quad \left| \frac{5^{2x} - 1}{5^{2x} + 1} \right| < 1$$

$$\begin{cases} \frac{5^{2x} - 1}{5^{2x} + 1} > -1 \\ \frac{5^{2x} - 1}{5^{2x} + 1} < 1 \end{cases} \quad \begin{cases} \frac{5^{2x} - 1 + 5^{2x} + 1}{5^{2x} + 1} > 0 \\ \frac{5^{2x} - 1 - 5^{2x} - 1}{5^{2x} + 1} < 0 \end{cases} \quad \forall x \in \mathbb{R}$$

$$30. \quad 4^x - 3 \cdot 2^x + 2 < 0$$

$$\text{Pongo: } 2^x = t \Rightarrow t^2 - 3t + 2 < 0 \Rightarrow 1 < t < 2 \Rightarrow$$

$$2^0 < 2^x < 2^1 \Rightarrow 0 < x < 1$$

$$31. \quad \left(\frac{1}{3}\right)^x - 3^{x+1} \geq 0$$

$$3^{-x} \geq 3^{x+1} \Rightarrow -x \geq x+1 \Rightarrow -2x \geq 1 \Rightarrow x \leq -\frac{1}{2}$$

$$32. \quad \left(\frac{1}{2}\right)^x - \left(\frac{1}{2}\right)^{-x} \geq 0$$

$$\text{Pongo: } \left(\frac{1}{2}\right)^x = t \Rightarrow t - \frac{1}{t} \geq 0 \Rightarrow t^2 - 1 \geq 0 \Rightarrow$$

$$t \leq -1 \vee t \geq 1 \Rightarrow \left(\frac{1}{2}\right)^x \geq 1 \Rightarrow x \leq 0$$

$$33. \quad 5^{x+2} + 25^{x+1} > 750$$

$$\text{Pongo: } 5^x = t \Rightarrow 25t + 25t^2 > 750 \Rightarrow t^2 + t - 30 > 0$$

$$t < -6 \vee t > 5 \Rightarrow 5^x > 5 \Rightarrow x > 1$$

$$34. \quad 2^{x+1} + \frac{8}{2^x} \geq 17$$

$$\text{Pongo: } 2^x = t \Rightarrow 2t + \frac{8}{t} \geq 17 \Rightarrow 2t^2 - 17t + 8 \geq 0$$

$$t \leq \frac{1}{2} \vee t \geq 8 \Rightarrow 2^x \leq 2^{-1} \vee 2^x \geq 2^3 \Rightarrow x \leq -1 \vee x \geq 3$$

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$$35. \quad 2 \left(\frac{1}{2} \right)^x - 2^x \leq 1$$

$$\text{Pongo: } 2^x = t \Rightarrow \frac{2}{t} - t \leq 1 \Rightarrow t^2 + t - 2 \geq 0 \Rightarrow$$

$$t \leq -2 \vee t \geq 1 \Rightarrow 2^x \geq 1 \Rightarrow x \geq 0$$

$$36. \quad \frac{7^{2x} - 7^x}{7^{2x} + 7^x} \geq 0$$

$$\frac{7^x (7^x - 1)}{7^{2x} + 7^x} \geq 0 \Rightarrow 7^x - 1 \geq 0 \Rightarrow 7^x \geq 7^0 \Rightarrow x \geq 0$$

$$37. \quad \frac{e^{2x}}{e^x - 1} < 0$$

$$e^x - 1 < 0 \Rightarrow e^x < e^0 \Rightarrow x < 0$$

$$38. \quad \frac{8 - 2^x}{2^{x+1}} \leq 0$$

$$8 - 2^x \leq 0 \Rightarrow -2^x \leq -8 \Rightarrow 2^x \geq 2^3 \Rightarrow x \geq 3$$

$$39. \quad \frac{4^x - 1}{x - 1} \leq 0$$

$$N \geq 0: 4^x - 1 \geq 0 \Rightarrow x \geq 0$$

$$D > 0: x - 1 > 0 \Rightarrow x > 1$$

$$0 \leq x < 1$$

$$40. \quad \frac{e^{x+1} - e^x}{e^{5x} - 1} \leq 0$$

$$N \geq 0: e^{x+1} - e^x \geq 0 \Rightarrow x + 1 \geq x \quad \forall x \in \mathbb{R}$$

$$D > 0: e^{5x} - 1 > 0 \Rightarrow x > 0$$

$$x < 0$$

$$41. \quad \frac{9^x + 3^x}{3^{2x} - 1} < 0$$

$$N > 0: \forall x \in \mathbb{R}$$

$$D > 0: 3^{2x} - 1 > 0 \Rightarrow x > 0$$

$$x < 0$$