

Disequazioni esponenziali

55. $3^{2x+1} - 10 \cdot 3^x + 3 \leq 0$

Pongo: $3^x = t \Rightarrow 3t^2 - 10t + 3 \leq 0 \Rightarrow \frac{1}{3} \leq t \leq 3$

$\frac{1}{3} \leq 3^x \leq 3 \Rightarrow 3^{-1} \leq 3^x \leq 3 \Rightarrow -1 \leq x \leq 1$

56. $\frac{\left(\frac{2}{3}\right)^{x-1} - 1}{\sqrt{2} - \sqrt[3]{2^{x-1}}} < 0$

$N > 0: \left(\frac{2}{3}\right)^{x-1} > 1 \Rightarrow x - 1 < 0 \Rightarrow x < 1$

$D > 0: 2^{\frac{1}{2}} > 2^{\frac{x-1}{3}} \Rightarrow \frac{x-1}{3} < \frac{1}{2} \Rightarrow 2x - 2 < 3 \Rightarrow x < \frac{5}{2}$ $1 < x < \frac{5}{2}$

57. $\frac{e^{\sqrt{x}} - e^{1-x}}{2^{x+1} - \sqrt{2}} \geq 0$

$N \geq 0: e^{\sqrt{x}} \geq e^{1-x} \Rightarrow \sqrt{x} \geq 1 - x \Rightarrow$

$\begin{cases} x \geq 0 \\ 1 - x < 0 \end{cases} \cup \begin{cases} 1 - x \geq 0 \\ x \geq 1 - 2x + x^2 \end{cases}$

$\begin{cases} x \geq 0 \\ x > 1 \end{cases} \cup \begin{cases} x \leq 1 \\ x^2 - 3x + 1 \leq 0 \end{cases}$

$x > 1 \cup \frac{3 - \sqrt{5}}{2} \leq x \leq 1 \Rightarrow x \geq \frac{3 - \sqrt{5}}{2}$

$D > 0: 2^{x+1} > 2^{\frac{1}{2}} \Rightarrow x + 1 > \frac{1}{2} \Rightarrow x > -\frac{1}{2}$

$x < -\frac{1}{2} \vee x \geq \frac{3 - \sqrt{5}}{2}$ ma $x \geq 0$ perciò: $x \geq \frac{3 - \sqrt{5}}{2}$

58. $\frac{3^{-x+4} (x^4 - 16)}{5^{1-x} - 5^{2x}} \leq 0$

$\frac{3^{-x+4} (x^2 + 4) (x + 2) (x - 2)}{5^{1-x} - 5^{2x}} \leq 0$

$N \geq 0: 3^{-x+4} (x^2 + 4) (x + 2) (x - 2) \geq 0 \Rightarrow x \leq -2 \vee x \geq 2$

$D > 0: 5^{1-x} > 5^{2x} \Rightarrow 1 - x > 2x \Rightarrow x < \frac{1}{3}$ $-2 < x \leq \frac{1}{3} \vee x \geq 2$

$$59. \quad 5^{4-x} - \left(\frac{1}{5}\right)^{\sqrt{x+1}} \leq 0$$

$$5^{4-x} \leq 5^{-\sqrt{x+1}} \Rightarrow 4-x \leq -\sqrt{x+1} \Rightarrow \sqrt{x+1} \leq x-4$$

$$\begin{cases} x \geq -1 \\ x > 4 \\ x+1 \leq x^2 - 8x + 16 \end{cases} \quad \begin{cases} x \geq -1 \\ x > 4 \\ x \leq \frac{9-\sqrt{21}}{2} \vee x \geq \frac{9+\sqrt{21}}{2} \end{cases}$$

$$x \geq \frac{9+\sqrt{21}}{2}$$

$$60. \quad (e^{2x} - e^{\sqrt{x+2}})(2^{1-x} - 2^x) \leq 0$$

$$e^{2x} - e^{\sqrt{x+2}} \geq 0 \Rightarrow e^{2x} \geq e^{\sqrt{x+2}}$$

$$\sqrt{x+2} \leq 2x \Rightarrow \begin{cases} x+2 \geq 0 \\ 2x \geq 0 \\ x+2 \leq 4x^2 \end{cases}$$

$$\begin{cases} x \geq -2 \\ x \geq 0 \\ 4x^2 - x - 2 \geq 0 \end{cases} \Rightarrow x \geq \frac{1+\sqrt{33}}{8}$$

$$2^{1-x} - 2^x \geq 0 \Rightarrow 2^{1-x} \geq 2^x \Rightarrow 1-x \geq x$$

$$2x \leq 1 \Rightarrow x \leq \frac{1}{2}$$

Tenendo presente la condizione di esistenza: $x \geq -2$

$$-2 \leq x \leq \frac{1}{2} \vee x \geq \frac{1+\sqrt{33}}{8}$$

$$61. \quad (e^{1-\sqrt{x}} - 1) \left[\left(\frac{1}{2}\right)^{\frac{4+x}{x}} - 4 \right] \leq 0$$

$$e^{1-\sqrt{x}} - 1 \geq 0 \Rightarrow e^{1-\sqrt{x}} \geq 1 \Rightarrow 1-\sqrt{x} \geq 0$$

$$\sqrt{x} \leq 1 \Rightarrow 0 \leq x \leq 1$$

$$\left(\frac{1}{2}\right)^{\frac{4+x}{x}} - 4 \geq 0 \Rightarrow \left(\frac{1}{2}\right)^{\frac{4+x}{x}} \geq \left(\frac{1}{2}\right)^{-2} \Rightarrow$$

$$\frac{4+x}{x} \leq -2 \Rightarrow \frac{3x+4}{x} \leq 0 \Rightarrow -\frac{4}{3} \leq x < 0$$

$$0 < x \leq 1$$

Disequazioni esponenziali

$$62. \frac{e^{\sqrt{1-x}} - e^{2-2x}}{(0,1)^{x^2-3} - 10^{2x}} \leq 0$$

$$N \geq 0: e^{\sqrt{1-x}} \geq e^{2-2x} \Rightarrow \sqrt{1-x} \geq 2-2x$$

$$\begin{cases} 1-x \geq 0 \\ 2-2x < 0 \end{cases} \cup \begin{cases} 2-2x \geq 0 \\ 1-x \geq 4-8x+4x^2 \end{cases}$$

$$\begin{cases} x \leq 1 \\ x > 1 \end{cases} \cup \begin{cases} x \leq 1 \\ 4x^2 - 7x + 3 \leq 0 \end{cases}$$

$$\frac{3}{4} \leq x \leq 1$$

$$D > 0: (0,1)^{x^2-3} > (0,1)^{-2x} \Rightarrow x^2 - 3 < -2x$$

$$x^2 + 2x - 3 < 0 \Rightarrow -3 < x < 1$$

$$-3 < x \leq \frac{3}{4}$$

$$63. \frac{2^{-2x} - 2^{x+1}}{2^{x^2} - 16} \geq 0$$

$$N \geq 0: 2^{-2x} \geq 2^{x+1} \Rightarrow -2x \geq x+1 \Rightarrow x \leq -\frac{1}{3}$$

$$D > 0: 2^{x^2} > 2^4 \Rightarrow x^2 > 4 \Rightarrow x < -2 \vee x > 2$$

$$x < -2 \vee -\frac{1}{3} \leq x < 2$$

$$64. e^{4x} > 4$$

$$4x > \ln 4 \Rightarrow x > \ln \sqrt[4]{4} \Rightarrow x > \ln \sqrt{2}$$

$$65. 2e^{x+3} > 5$$

$$e^{x+3} > \frac{5}{2} \Rightarrow x+3 > \ln \frac{5}{2} \Rightarrow x > \ln \frac{5}{2} - 3$$

$$66. 5^{2x} - 2 \cdot 5^x - 3 \geq 0$$

$$\text{Pongo: } 5^x = t \Rightarrow t^2 - 2t - 3 \geq 0 \Rightarrow t_{1,2} = \frac{1 \pm \sqrt{1+3}}{1} \begin{cases} 3 \\ -1 \end{cases} \Rightarrow$$

$$t \leq -1 \vee t \geq 3 \Rightarrow 5^x \leq -1 \vee 5^x \geq 3 \Rightarrow x \geq \log_5 3$$