

67. $(e^x - 2)(-e^{-x} - 1) \geq 0$

$F_1 \geq 0: e^x - 2 \geq 0 \Rightarrow x \geq \ln 2$

$F_2 \geq 0: -e^{-x} - 1 \geq 0 \text{ imp.} \Rightarrow$

$x \leq \ln 2$

68. $2\left(\frac{1}{9}\right)^{\frac{x+1}{2}} - \left(\frac{1}{9}\right)^x + \frac{5}{3} \geq 0$

Pongo: $\left(\frac{1}{9}\right)^{\frac{x}{2}} = t \Rightarrow 2\left(\frac{1}{9}\right)^{\frac{x}{2}} t - t^2 + \frac{5}{3} \geq 0 \Rightarrow \frac{2}{3}t - t^2 + \frac{5}{3} \geq 0$

$3t^2 - 2t - 5 \leq 0 \Rightarrow t_{1,2} = \frac{1 \pm \sqrt{1+15}}{3} \left\langle \begin{matrix} \frac{5}{3} \\ -1 \end{matrix} \right.$

$-1 \leq t \leq \frac{5}{3} \Rightarrow -1 \leq \left(\frac{1}{3}\right)^x \leq \frac{5}{3} \Rightarrow x \geq \log_{\frac{1}{3}} \frac{5}{3}$

69. $|3^{2x} - 3^x| < 2$

$\begin{cases} 3^{2x} - 3^x < 2 \\ 3^{2x} - 3^x > -2 \end{cases} \Rightarrow \begin{cases} 3^{2x} - 3^x - 2 < 0 \\ 3^{2x} - 3^x + 2 > 0 \end{cases}$

Pongo: $3^x = t \Rightarrow \begin{cases} t^2 - t - 2 < 0 \\ t^2 - t + 2 > 0 \end{cases} \Rightarrow \begin{cases} -1 < t < 2 \\ \forall x \in R \end{cases} \Rightarrow -1 < 3^x < 2$

$3^x < 2 \Rightarrow x < \log_3 2$

70. $\frac{3 \cdot 2^{2x+2} - 12}{2^x} \leq 2^x + 7 \cdot 2^{2x} - 7 - 2^{3x}$

Pongo: $2^x = t \Rightarrow \frac{12t^2 - 12}{t} \leq t + 7t^2 - 7 - t^3$

Faccio il minimo comune multiplo e posso semplificare il denominatore, visto che è 2^x , che è sempre positivo:

$12t^2 - 12 - t^2 - 7t^3 + 7t + t^4 \leq 0 \Rightarrow t^4 - 7t^3 + 11t^2 + 7t - 12 \leq 0$

$(t-1)(t+1)(t-3)(t-4) \leq 0 \Rightarrow -1 \leq t \leq 1 \vee 3 \leq t \leq 4$

$-1 \leq 2^x \leq 1 \vee 3 \leq 2^x \leq 4 \Rightarrow x \leq 0 \vee \log_2 3 \leq x \leq 2$

71. $2e^{3x} - 9e^{2x} + e^x + 12 \leq 0$

Pongo: $e^x = t \Rightarrow 2t^3 - 9t^2 + t + 12 \leq 0$

$$(t + 1)(t - 4)(2t - 3) \leq 0 \Rightarrow t \leq -1 \vee \frac{3}{2} \leq t \leq 4 \Rightarrow$$

$$e^x \leq -1 \vee \frac{3}{2} \leq e^x \leq 4 \Rightarrow \ln \frac{3}{2} \leq x \leq \ln 4$$

72. $\frac{3e^{2x}}{4 - e^x} \geq 1$

$$\frac{3e^{2x}}{4 - e^x} - 1 \geq 0 \Rightarrow \frac{3e^{2x} - 4 + e^x}{4 - e^x} \geq 0$$

Pongo: $e^x = t \Rightarrow N \geq 0: 3t^2 + t - 4 \geq 0 \Rightarrow t \leq -\frac{4}{3} \vee t \geq 1 \Rightarrow$

$$e^x \leq -\frac{4}{3} \vee e^x \geq 1 \Rightarrow x \geq 0$$

$D > 0: 4 - e^x > 0 \Rightarrow e^x < 4 \Rightarrow x < \ln 4$

$0 \leq x < \ln 4$

73. $\left(2^{x^2} - \frac{1}{3}\right)(5^{3x} - 6 \cdot 5^{2x} + 3 \cdot 5^x + 10) \leq 0$

$$F_1 \geq 0 \Rightarrow 2^{x^2} - \frac{1}{3} \geq 0 \Rightarrow x^2 \geq \log_2 \frac{1}{3} \Rightarrow x^2 \geq -\log_2 3 \quad \forall x \in \mathbb{R}$$

$F_2 \geq 0 \Rightarrow$ Pongo: $5^x = t \Rightarrow t^3 - 6t^2 + 3t + 10 \geq 0 \Rightarrow$

$$(t + 1)(t - 2)(t - 5) \geq 0 \Rightarrow -1 \leq t \leq 2 \vee t \geq 5$$

$$-1 \leq 5^x \leq 2 \vee 5^x \geq 5 \Rightarrow x \leq \log_5 2 \vee x \geq 1 \Rightarrow \log_5 2 \leq x \leq 1$$

74. $3^{\sqrt{x+1}} - 9^{\sqrt{x}} + 4 \geq 0$

Pongo: $3^{\sqrt{x}} = t \Rightarrow 3t - t^2 + 4 \geq 0 \Rightarrow t^2 - 3t - 4 \leq 0 \Rightarrow -1 \leq t \leq 4$

$$-1 \leq 3^{\sqrt{x}} \leq 4 \Rightarrow 3^{\sqrt{x}} \leq 4 \Rightarrow \sqrt{x} \leq \log_3 4 \Rightarrow 0 \leq x \leq \log_3^2 4$$

75. $\frac{2 - 5^x}{2 \cdot 5^x - 2} + \frac{2}{25^x - 5^x} \leq \frac{3 - 5^x}{5^x - 1}$

Pongo: $5^x = t \Rightarrow \frac{2 - t}{2t - 2} + \frac{2}{t^2 - t} - \frac{3 - t}{t - 1} \leq 0 \Rightarrow$

$$\frac{2t - t^2 + 4 - 6t + 2t^2}{2t(t - 1)} \leq 0 \Rightarrow \frac{t^2 - 4t + 4}{2t(t - 1)} \leq 0 \Rightarrow \frac{(t - 2)^2}{2t(t - 1)} \leq 0$$

$$0 < t < 1 \vee t = 2 \Rightarrow 0 < 5^x < 1 \vee 5^x = 2 \Rightarrow x < 0 \vee x = \log_5 2$$

Disequazioni esponenziali

$$76. \begin{cases} (e^{\sqrt{2x+3}} - e^x)(e^{2x} - e^x - 2) \leq 0 \\ (e^{\sqrt{2x+3}} - e^x)(e^x - 2)(e^x + 1) \leq 0 \end{cases}$$

$$F_1 \geq 0: e^{\sqrt{2x+3}} - e^x \geq 0 \Rightarrow \sqrt{2x+3} \geq x \Rightarrow$$

$$\begin{cases} 2x+3 \geq 0 \\ x < 0 \end{cases} \vee \begin{cases} 2x+3 \geq x^2 \\ x \geq 0 \end{cases} \Rightarrow -\frac{3}{2} \leq x \leq 3$$

$$F_2 \geq 0: e^x - 2 \geq 0 \Rightarrow x \geq \ln 2$$

$$F_3 \geq 0: e^x + 1 \geq 0 \Rightarrow \forall x \in \mathbb{R}$$

$$-\frac{3}{2} \leq x \leq \ln 2 \vee x \geq 3$$

$$77. 18^{2x^2-3x+1} > 1$$

$$18^{2x^2-3x+1} > 18^0 \Rightarrow 2x^2 - 3x + 1 > 0$$

$$x_{1,2} = \frac{3 \pm \sqrt{9-8}}{4} \begin{cases} 1 \\ \frac{1}{2} \end{cases} \Rightarrow x < \frac{1}{2} \vee x > 1$$

$$78. (5^x)^x < 5$$

$$5^{x^2} < 5 \Rightarrow x^2 < 1 \Rightarrow -1 < x < 1$$

$$79. \left(\frac{1}{2}\right)^{-x-1} > \left(\frac{1}{2}\right)^{x+1}$$

$$-x-1 < x+1 \Rightarrow -2x-2 < 0 \Rightarrow 2x+2 > 0 \Rightarrow x > -1$$

$$80. 4^x - 5 \cdot 2^x + 4 > 0$$

$$\text{Pongo: } 2^x = t \Rightarrow t^2 - 5t + 4 > 0 \Rightarrow t_{1,2} = \frac{5 \pm \sqrt{25-16}}{2} \begin{cases} 4 \\ 1 \end{cases}$$

$$t < 1 \vee t > 4 \Rightarrow x < 0 \vee x > 2$$

$$81. 5^{x+1} < 25^{x-1}$$

$$5^{x+1} < 5^{2x-2} \Rightarrow x+1 < 2x-2 \Rightarrow -x < -3 \Rightarrow x > 3$$

$$82. \left(\frac{1}{3}\right)^{x+3} > 9$$

$$\left(\frac{1}{3}\right)^{x+3} > \left(\frac{1}{3}\right)^{-2} \Rightarrow x+3 < -2 \Rightarrow x < -5$$