PRODOTTI NOTEVOLI

PRODOTTO DELLA SOMMA DI DUE MONOMI PER LA LORO DIFFERENZA

1. $(a - 2b)(a + 2b) = a^2 - 4b^2$ $\qquad$ $\left(\dfrac{1}{2}a + 3b\right)\left(-\dfrac{1}{2}a + 3b\right) = 9b^2 - \dfrac{1}{4}a^2$

2. $(2a + 1)(2a - 1) = 4a^2 - 1$ $\qquad$ $(-3x - y)(-3x + y) = 9x^2 - y^2$

3. $(2x - 4y)(-2x - 4y) = 16y^2 - 4x^2$ $\qquad$ $(2 - x)(x + 2) = 4 - x^2$

4. $(-x - y)(-x + y) = x^2 - y^2$ $\qquad$ $(-x^2 - y^2)(x^2 - y^2) = -x^4 + y^4$

5. $(-3x - 2y)(3x - 2y) = 4y^2 - 9x^2$ $\qquad$ $(x - 0{,}1)(0{,}1 + x) = x^2 - \dfrac{1}{100}$

6. $(-a - b)(a - b)(a^2 + b^2) = b^4 - a^4$ $\qquad$ $(2x^n + y^3)(-2x^n + y^3) = -4x^{2n} + y^6$

7. $(a^n - 2)(a^n + 2) = a^{2n} - 4$ $\qquad$ $(x^{n+1} - 1)(x^{n+1} + 1) = x^{2n+2} - 1$

8. $(1 - a^n)(a^n + 1) = 1 - a^{2n}$ $\qquad$ $(x^{2n+3} - y^m)(x^{2n+3} + y^m) = x^{4n+6} - y^{2m}$

9. $(2a^m - b^{3n})(2a^m + b^{3n}) = 4a^{2m} - b^{6n}$ $\qquad$ $(x^n - y)(-x^n - y) = -x^{2n} + y^2$

10. $(a^{n-m} + a)(a^{n-m} - a) = a^{2n-2m} - a^2$

11. $(a + b)(a - b)(a^2 + b^2)(a^4 + b^4) = (a^2 - b^2)(a^2 + b^2)(a^4 + b^4) = (a^4 - b^4)(a^4 + b^4) = a^8 - b^8$

12. $(x^2 + 2)(x^2 - 2)(x^4 + 4)(x^8 + 16) = (x^4 - 4)(x^4 + 4)(x^8 + 16) = (x^8 - 16)(x^8 + 16) = x^{16} - 256$

13. $\left(-\dfrac{1}{3}x - 2\right)\left(\dfrac{1}{3}x - 2\right)\left(\dfrac{1}{9}x^2 + 4\right) = \left(-\dfrac{1}{9}x^2 + 4\right)\left(\dfrac{1}{9}x^2 + 4\right) = -\dfrac{1}{81}x^4 + 16$

14. $[(a - b)(a + b)]^2 = (a^2 - b^2)^2 = a^4 - 2a^2b^2 + b^4$

15. $[(x - 3y)(x + 3y)]^2 = (x^2 - 9y^2)^2 = x^4 - 18x^2y^2 + 81y^4$

16. $(a - 2)^2(a + 2)^2 = (a^2 - 4)^2 = a^4 - 8a^2 + 16$

17. $[(x^3 - 3ax^2)(x^3 + 3ax^2)]^2 = (x^6 - 9a^2x^4)^2 = x^{12} - 18a^2x^{10} + 81a^4x^8$

18. $\left[\left(-\dfrac{1}{3} - a^5\right)\left(-\dfrac{1}{3} + a^5\right)\right]^2 = \left(\dfrac{1}{9} - a^{10}\right)^2 = \dfrac{1}{81} - \dfrac{2}{9}a^{10} + a^{20}$

19. $(x^2 - y)^2(-x^2 - y)^2 = (-x^4 + y^2)^2 = x^8 - 2x^4y^2 + y^4$

20. $[x + (x^2 - 1)][x - (x^2 - 1)] = x^2 - (x^2 - 1)^2 = x^2 - (x^4 - 2x^2 + 1) = -x^4 + x^2 - 1$

21. $[(x + y) + 4][(x + y) - 4] = x^2 + 2xy + y^2 - 16$

22. $[x^2 + (y^2 - z)][x^2 - (y^2 - z)] = x^4 - (y^4 - 2y^2z + z^2) = x^4 - y^4 + 2y^2z - z^2$

23. $(x^2 + y^2 - z)(x^2 + y^2 + z) = (x^2 + y^2)^2 - z^2 = x^4 + 2x^2y^2 + y^4 - z^2$

24. $(x^2 + 3xy - z^2)(x^2 - 3xy + z^2) = x^4 - (3xy - z^2)^2 = x^4 - 9x^2y^2 + 6xyz^2 - z^4$

25. $(x^2 + 3xy - z^2)(x^2 - 3xy - z^2) = (x^2 - z^2)^2 - 9x^2y^2 = x^4 - 2x^2z^2 + z^4 - 9x^2y^2$

26. $(x^2 + 3xy + z^2)(x^2 - 3xy - z^2) = x^4 - (3xy + z^2)^2 = x^4 - 9x^2y^2 - 6xyz^2 - z^4$

27. $(1 - 3x + y)(1 + 3x - y) = 1 - (3x - y)^2 = 1 - 9x^2 + 6xy - y^2$

28. $(a^2 - a + b^2)(a^2 + a - b^2) = a^4 - (a - b^2)^2 = a^4 - a^2 + 2ab^2 - b^4$

29. $\left(-\dfrac{2}{3} + a - \dfrac{b}{2}\right)\left(-\dfrac{b}{2} + \dfrac{2}{3} - a\right) = -\dfrac{b^2}{4} - \left(\dfrac{2}{3} - a\right)^2 = -\dfrac{b^2}{4} - \dfrac{4}{9} + \dfrac{4}{3}a - a^2$

30. $(a + a^2 + a^3)(a^3 - a^2 + a) = (a^3 + a)^2 - a^4 = a^6 + 2a^4 + a^2 - a^4 = a^6 + a^4 + a^2$

31. $[a + (b - c)][a - (b - c)] = a^2 - (b - c)^2 = a^2 - (b^2 - 2bc + c^2) = a^2 - b^2 + 2bc - c^2$

32. $[(a^2 - 3c) + 1][(a^2 - 3c) - 1] = (a^2 - 3c)^2 - 1 = a^4 - 6a^2c + 9c^2 - 1$

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33. $3(a^2 + 2b)(a^2 - 2b) - 2(2a^2 + 3b)(2a^2 - 3b) = 3(a^4 - 4b^2) - 2(4a^4 - 9b^2) =$
$= 3a^4 - 12b^2 - 8a^4 + 18b^2 = -5a^4 + 6b^2$

34. $(0,2x - y^3)(0,2x + y^3) - (0,1x + 3y^3)(0,1x - 3y^3) = 0,04x^2 - y^6 - (0,01x^2 - 9y^6) =$
$= 0,04x^2 - y^6 - 0,01x^2 + 9y^6 = 0,03x^2 + 8y^6$

35. $(2ab + 3a^2b^3)(2ab - 3a^2b^3) : (-3ab^2) = (4a^2b^2 - 9a^4b^6) : (-3ab^2) = -\dfrac{4}{3}a + 3a^3b^4$

36. $(3a^3b^2 - 6a^4b^3) : (-3a^3b^2) \cdot (1 + 2ab) = (-1 + 2ab)(1 + 2ab) = 4a^2b^2 - 1$

37. $(1 - 3x^2)[(4x^3 + 12x^5) : (-4x^3)] = (1 - 3x^2)(-1 - 3x^2) = 9x^4 - 1$

38. $(2a - 5b)(2a + 5b) + (a - 3b)^2 + (a + 4b)(-a + 4b) =$
$= 4a^2 - 25b^2 + a^2 - 6ab + 9b^2 - a^2 + 16b^2 = 4a^2 - 6ab$

39. $(x + 2y)(x - 2y) - (2x - 3y)^2 + (-x - 5y)^2 =$
$= x^2 - 4y^2 - 4x^2 + 12xy - 9y^2 + x^2 + 10xy + 25y^2 = -2x^2 + 22xy + 12y^2$

40. $(a^2 - 1)(1 + a^2) - (a^2 - 2)^2 + [(a + 1)(a - 1)]^2 - (a^2 - 2)(a^2 + 2) =$
$= a^4 - 1 - (a^4 - 4a^2 + 4) + (a^2 - 1)^2 - (a^4 - 4) =$
$= a^4 - 1 - a^4 + 4a^2 - 4 + a^4 - 2a^2 + 1 - a^4 + 4 = 2a^2$

41. $a^2(x - 1)^2 + (ax + 3)(3 - ax) + 2x\left(a - \dfrac{3}{2}\right)^2 - (a - 3)^2 =$

$= a^2(x^2 - 2x + 1) - a^2x^2 + 9 + 2x\left(a^2 - 3a + \dfrac{9}{4}\right) - (a^2 - 6a + 9) =$

$= a^2x^2 - 2a^2x + a^2 - a^2x^2 + 9 + 2a^2x - 6ax + \dfrac{9}{2}x - a^2 + 6a - 9 = \dfrac{9}{2}x - 6ax + 6a$

42. $9x^2\left(\dfrac{1}{3}y - x^2\right)\left(\dfrac{1}{3}y + x^2\right) + (3x^3 + 2xy)^2 - 5(-xy)^2 =$

$= 9x^2\left(\dfrac{1}{9}y^2 - x^4\right) + 9x^6 + 12x^4y + 4x^2y^2 - 5(x^2y^2) =$

$= x^2y^2 - 9x^6 + 9x^6 + 12x^4y + 4x^2y^2 - 5x^2y^2 = 12x^4y$

43. $[x^4(3x + 4)^2 : (-2x)^3 + 2x] : (-3x)^2 - x\left(-\dfrac{1}{2}\right)^3 + \dfrac{1}{3} =$

$= [x^4(9x^2 + 24x + 16) : (-8x^3) + 2x] : (9x^2) - x\left(-\dfrac{1}{8}\right) + \dfrac{1}{3} =$

$= [(9x^6 + 24x^5 + 16x^4) : (-8x^3) + 2x] : (9x^2) + \dfrac{1}{8}x + \dfrac{1}{3} =$

$$= \left[-\frac{9}{8} x^3 - 3x^2 - 2x + 2x \right] : (9x^2) + \frac{1}{8} x + \frac{1}{3} = -\frac{1}{8} x - \frac{1}{3} + \frac{1}{8} x + \frac{1}{3} = 0$$

44. $\left[(a^8 - a^{12} - a^{16}) : \left(-\frac{1}{2} a^8 \right) + 2 \right]^2 : [-2(-a)^4]^2 - (-a^2)^2 (a^4 + 2) =$

$$= (-2 + 2a^4 + 2a^8 + 2)^2 : (-2a^4)^2 - a^4 (a^4 + 2) =$$
$$= (-1 - a^4)^2 - a^8 - 2a^4 = 1 + 2a^4 + a^8 - a^8 - 2a^4 = 1$$

45. $[a(a-b) - b(b-a)]^2 - 2(a-b)^2 (a+b)^2 + a^2 (a^2 - 2b^2) =$
$$= (a^2 - ab - b^2 + ab)^2 - 2(a^2 - b^2)^2 + a^4 - 2a^2 b^2 = (a^2 - b^2)^2 - 2(a^2 - b^2)^2 + a^4 - 2a^2 b^2 =$$
$$= -(a^2 - b^2)^2 + a^4 - 2a^2 b^2 = -a^4 + 2a^2 b^2 - b^4 + a^4 - 2a^2 b^2 = -b^4$$

46. $(x^2 + x + 1)(x^2 - x + 1) - x^2 (x^2 + 1) =$
$$= (x^2 + 1)^2 - x^2 - x^4 - x^2 = x^4 + 2x^2 + 1 - 2x^2 - x^4 = 1$$

47. $\left\{ (3x^2 + xy - xz)(3x^2 - xy - xz) : (3x^2) + \frac{1}{3}(y-z)(y+z) \right\} : (-x) =$

$$= \left\{ (9x^4 - 6x^3 z + x^2 z^2 - x^2 y^2) : (3x^2) + \frac{1}{3}(y^2 - z^2) \right\} : (-x) =$$

$$= \left\{ 3x^2 - 2xz + \frac{1}{3} z^2 - \frac{1}{3} y^2 + \frac{1}{3} y^2 - \frac{1}{3} z^2 \right\} : (-x) = -3x + 2z$$

48. $[(3a - 2b)^2 - (2b + 9a^2)^2 + 12ab] : [(3a - 2b)(3a + 2b) + 4b^2] =$
$$= (9a^2 - 12ab + 4b^2 - 4b^2 - 36a^2 b - 81a^4 + 12ab) : (9a^2 - 4b^2 + 4b^2) =$$
$$= (9a^2 - 36a^2 b - 81a^4) : (9a^2) = 1 - 4b - 9a^2$$

49. $(1 - x - y + x^2)(1 - x + x^2 + y) + (2x^2 - 1)(2x^2 + 1) + 2x(1 + x)^2 =$
$$= (1 - x + x^2)^2 - y^2 + 4x^4 - 1 + 2x(1 + 2x + x^2) =$$
$$= 1 + x^2 + x^4 - 2x + 2x^2 - 2x^3 - y^2 + 4x^4 - 1 + 2x + 4x^2 + 2x^3 = 5x^4 + 7x^2 - y^2$$

50. $3y(x-y)^2 + (x-y)(x+y)(2x-y) - (x-2y)(x+2y)(2x-y) =$
$$= 3y(x^2 - 2xy + y^2) + (x^2 - y^2)(2x-y) - (x^2 - 4y^2)(2x-y) =$$
$$= 3x^2 y - 6xy^2 + 3y^3 + x^2(2x-y) - y^2(2x-y) - x^2(2x-y) + 4y^2(2x-y) =$$
$$= 3x^2 y - 6xy^2 + 3y^3 + 3y^2(2x-y) = 3x^2 y - 6xy^2 + 3y^3 + 6xy^2 - 3y^3 = 3x^2 y$$

CUBO DI UN BINOMIO

51. $(2a + 1)^3 = 8a^3 + 12a^2 + 1 + 6a$ $\qquad (2a + b)^3 = 8a^3 + 12a^2 b + b^3 + 6ab^2$

52. $\left(2x - \frac{1}{3} y \right)^3 = 8x^3 - \frac{1}{27} y^3 + \frac{2}{3} xy^2 - 4x^2 y$ $\qquad (ab + 1)^3 = a^3 b^3 + 3a^2 b^2 + 3ab + 1$

53. $[(ax^3 - x^2)(ax^3 + x^2)]^3 = (a^2 x^6 - x^4)^3 = a^6 x^{18} - 3a^4 x^{16} + 3a^2 x^{14} - x^{12}$

54. $[(1 - x^3)(1 + x^3)]^3 = (1 - x^6)^3 = 1 - 3x^6 + 3x^{12} - x^{18}$

55. $\left[\left(a - \frac{1}{3} b \right) \left(a + \frac{1}{3} b \right) \right]^3 = \left(a^2 - \frac{1}{9} b^2 \right)^3 = a^6 - \frac{1}{3} a^4 b^2 + \frac{1}{27} a^2 b^4 - \frac{1}{729} b^6$

56. $\left[\left(\dfrac{2}{3}x^2 - \dfrac{3}{2}y^3\right)\left(\dfrac{2}{3}x^2 + \dfrac{3}{2}y^3\right)\right]^3 = \left(\dfrac{4}{9}x^4 - \dfrac{9}{4}y^6\right)^3 = \dfrac{64}{729}x^{12} - \dfrac{4}{3}x^8 y^6 + \dfrac{27}{4}x^4 y^{12} - \dfrac{729}{64}y^{18}$

57. $[(1 + x^3)^3]^2 = (1 + 3x^3 + 3x^6 + x^9)^2 =$

    $= 1 + 9x^6 + 9x^{12} + x^{18} + 6x^3 + 6x^6 + 6x^9 + 18x^9 + 6x^{12} + 6x^{15} =$

    $= 1 + 15x^6 + 15x^{12} + x^{18} + 6x^3 + 24x^9 + 6x^{15}$

58. $[(2 - x^3)^3]^2 = (8 - 12x^3 + 6x^6 - x^9)^2 =$

    $= 64 + 144x^6 + 36x^{12} + x^{18} - 192x^3 + 96x^6 - 16x^9 - 144x^9 + 24x^{12} - 12x^{15} =$

    $= 64 + 240x^6 + 60x^{12} + x^{18} - 192x^3 - 160x^9 + - 12x^{15}$

59. $(a^3 - 1)^3 (a^3 + 1)^3 = (a^6 - 1)^3 = a^{18} - 3a^{12} + 3a^6 - 1$

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60. $(x - a)^3 - (2x + a)^3 + (2a - x)^3 =$

    $= x^3 - 3ax^2 + 3a^2 x - a^3 - 8x^3 - 12ax^2 - 6a^2 x - a^3 + 8a^3 - 12a^2 x + 6ax^2 - x^3 =$

    $= -9ax^2 - 15a^2 x + 6a^3 - 8x^3$

61. $(a - 3b)^3 + (2a + 3b)^3 + (a - 2b)(a^2 + b^2) =$

    $= a^3 - 9a^2 b + 27ab^2 - 27b^3 + 8a^3 + 36a^2 b + 54ab^2 + 27b^3 + a^3 + ab^2 - 2a^2 b - 2b^3 =$

    $= 10a^3 + 25a^2 b + 82ab^2 - 2b^3$

62. $(2a + 3y)^3 - (2a - 3y)^3 - 54y^3 =$

    $8a^3 + 36a^2 y + 54ay^2 + 27y^3 - 8a^3 + 36a^2 y - 54ay^2 + 27y^3 - 54y^3 = 72a^2 y$

63. $(a + 3)^3 + 2(a^2 + 3)(a - 3) + (a - 3)^3 =$

    $= a^3 + 9a^2 + 27a + 27 + 2(a^3 - 3a^2 + 3a - 9) + a^3 - 9a^2 + 27a - 27 =$

    $= 2a^3 + 54a + 2a^3 - 6a^2 + 6a - 18 = 4a^3 - 6a^2 + 60a - 18$

64. $\left(\dfrac{1}{2} - x^2\right)^3 - (2x^2 - 1)^3 - (x - 2)^3 (x + 2)^3 =$

    $= \dfrac{1}{8} - \dfrac{3}{4}x^2 + \dfrac{3}{2}x^4 - x^6 - (8x^6 - 12x^4 + 6x^2 - 1) - (x^2 - 4)^3 =$

    $= \dfrac{1}{8} - \dfrac{3}{4}x^2 + \dfrac{3}{2}x^4 - x^6 - 8x^6 + 12x^4 - 6x^2 + 1 - x^6 + 12x^4 - 48x^2 + 64 =$

    $= \dfrac{521}{8} - \dfrac{219}{4}x^2 + \dfrac{51}{2}x^4 - 10x^6$