37. $\dfrac{1}{2} \ln x = \ln 3 + \ln (6 - \sqrt{x + 16})$

$c.a.: \begin{cases} x > 0 \\ 6 - \sqrt{x + 16} > 0 \\ x + 16 \geq 0 \end{cases} \Rightarrow \begin{cases} x > 0 \\ 36 > x + 16 \\ x \geq -16 \end{cases} \Rightarrow \begin{cases} x > 0 \\ x < 20 \\ x \geq -16 \end{cases} \Rightarrow 0 < x < 20$

$\ln \sqrt{x} = \ln 3(6 - \sqrt{x + 16}) \Rightarrow \sqrt{x} = 18 - 3\sqrt{x + 16} \Rightarrow 18 - \sqrt{x} = 3\sqrt{x + 16}$

$324 - 36\sqrt{x} + x = 9x + 144 \Rightarrow 2x - 45 = -36\sqrt{x} \Rightarrow 4x^2 - 261x + 2025 = 0$

$x_{1,2} = \dfrac{261 \pm 189}{8} \quad \begin{cases} \dfrac{225}{4} \text{ non acc.} \\ 9 \end{cases} \Rightarrow \boxed{x = 9}$

38. $4\left(\log_{\frac{1}{2}} x\right)^2 - 5 \log_{\frac{1}{2}} x + 1 = 0$

$c.a.: x > 0: \quad$ pongo: $\log_{\frac{1}{2}} x = t \Rightarrow 4t^2 - 5t + 1 = 0$

$t_{1,2} = \dfrac{5 \pm \sqrt{25 - 16}}{8} \Rightarrow \begin{cases} 1 \Rightarrow \log_{\frac{1}{2}} x = 1 \Rightarrow x = \dfrac{1}{2} \\ \dfrac{1}{4} \Rightarrow \log_{\frac{1}{2}} x = \dfrac{1}{4} \Rightarrow x = \sqrt[4]{\dfrac{1}{2}} \end{cases}$ $\boxed{x = \dfrac{1}{2}; \quad x = \sqrt[4]{\dfrac{1}{2}}}$

39. $(\log_3 x)^2 + \log_3 x - 12 = 0$

$c.a.: x > 0: \quad$ pongo: $\log_3 x = t \Rightarrow t^2 + t - 12 = 0$

$t_{1,2} = \dfrac{-1 \pm \sqrt{1 + 48}}{2} \Rightarrow \begin{cases} 3 \Rightarrow \log_3 x = 3 \Rightarrow x = 27 \\ -4 \Rightarrow \log_3 x = -4 \Rightarrow x = \dfrac{1}{81} \end{cases}$ $\boxed{x = 27; \quad x = \dfrac{1}{81}}$

40. $\log_a^3 x - \log_a^2 x = 0$

$c.a.: x > 0: \quad$ Pongo: $\log_a x = t \Rightarrow t^3 - t^2 = 0 \Rightarrow t^2(t - 1) = 0 \Rightarrow$

$t = 0 \Rightarrow \log_a x = 0 \Rightarrow \boxed{x = 1}$

$t = 1 \Rightarrow \log_a x = 1 \Rightarrow \boxed{x = a}$

41. $\ln^2 x - \ln x - 2 = 0$

    $c.a.: x > 0:$ Pongo: $\ln x = t \;\Rightarrow\; t^2 - t - 2 = 0 \;\Rightarrow\; t_{1,2} = \dfrac{1 \pm \sqrt{1+8}}{2}$

    $t = 2 \;\Rightarrow\; \ln x = 2 \;\Rightarrow\; \boxed{x = e^2}$

    $t = -1 \;\Rightarrow\; \ln x = -1 \;\Rightarrow\; \boxed{x = \dfrac{1}{e}}$

42. $\ln^3 x - 4 \ln x = 0$

    $c.a.: x > 0:$ Pongo: $\ln x = t \;\Rightarrow\; t^3 - 4t = 0 \;\Rightarrow\; t\,(t^2 - 4) = 0 \;\Rightarrow$

    $t\,(t-2)\,(t+2) = 0$

    $t = 0 \;\Rightarrow\; \ln x = 0 \;\Rightarrow\; \boxed{x = 1}$

    $t = 2 \;\Rightarrow\; \ln x = 2 \;\Rightarrow\; \boxed{x = e^2}$

    $t = -2 \;\Rightarrow\; \ln x = -2 \;\Rightarrow\; \boxed{x = \dfrac{1}{e^2}}$

43. $\ln^2 x + \ln x - 6 = 0$

    $c.a.: x > 0:$ Pongo: $\ln x = t \;\Rightarrow\; t^2 + t - 6 = 0 \;\Rightarrow\; t_{1,2} = \dfrac{-1 \pm \sqrt{1+24}}{2}$

    $t = 2 \;\Rightarrow\; \ln x = 2 \;\Rightarrow\; \boxed{x = e^2}$

    $t = -3 \;\Rightarrow\; \ln x = -3 \;\Rightarrow\; \boxed{x = \dfrac{1}{e^3}}$

44. $\ln^2 x - \ln x^2 + 1 = 0$

    $c.a.: x > 0:$ $\ln^2 x - 2\ln x + 1 = 0 \;\Rightarrow\; (\ln x - 1)^2 = 0$

    $\ln x = 1 \;\Rightarrow\; \boxed{x = e}$

45. $(\ln x)(\ln x^2) + \ln x^3 - 9 = 0$

    $c.a.: x > 0:$ $(\ln x)(2 \ln x) + 3 \ln x - 9 = 0$ pongo: $\ln x = t \;\Rightarrow\; 2t^2 + 3t - 9 = 0$

    $t_{1,2} = \dfrac{-3 \pm \sqrt{9+72}}{4} \;\Rightarrow\; \begin{cases} \dfrac{3}{2} \;\Rightarrow\; \ln x = \dfrac{3}{2} \;\Rightarrow\; x = e^{\frac{3}{2}} \\[2mm] -3 \;\Rightarrow\; \ln x = -3 \;\Rightarrow\; x = e^{-3} \end{cases}$

    $\boxed{x = e\sqrt{e}\,;\;\; x = \dfrac{1}{e^3}}$

46.　$(\ln x)^3 - 4(\ln x)^2 - 29\ln x - 24 = 0$

　　$c.a.: x > 0:$　pongo: $\ln x = t \Rightarrow t^3 - 4t^2 - 29t - 24 = 0$

　　Applico la regola di Ruffini, con t = -1 e ottengo: $(t + 1)(t^2 - 5t - 24) = 0$

　　$t_{1,2} = \dfrac{5 \pm \sqrt{25 + 96}}{2} \Rightarrow \begin{cases} 8 \Rightarrow \ln x = 8 \Rightarrow x = e^8 \\ -3 \Rightarrow \ln x = -3 \Rightarrow x = e^{-3} \end{cases}$

　　$t_3 = -1 \Rightarrow \ln x = -1 \Rightarrow x = e^{-1}$

　　$\boxed{x = e^8; \quad x = \dfrac{1}{e^3}; \quad x = \dfrac{1}{e}}$

47.　$\dfrac{1 + \log x}{\log x - 1} - \dfrac{\log x + 3}{2 - 2\log x} = \dfrac{11}{2}$

　　$c.a.: x > 0:$　pongo: $\log x = t \Rightarrow \dfrac{1 + t}{t - 1} - \dfrac{t + 3}{2 - 2t} = \dfrac{11}{2}$

　　$\dfrac{1 + t}{t - 1} + \dfrac{t + 3}{2(t - 1)} = \dfrac{11}{2} \Rightarrow 2 + 2t + t + 3 = 11t - 11 \quad c.a.: t \neq 1$

　　$8t = 16 \Rightarrow t = 2 \Rightarrow \log x = 2 \Rightarrow \boxed{x = 100}$

48.　$\dfrac{4}{\log_9 x} - \left(2 - \dfrac{3}{\log_9 x}\right) - 2\left(1 - \dfrac{1}{\log_9 x}\right) = 14$

　　$c.a.: x > 0:$　pongo: $\log_9 x = t \Rightarrow \dfrac{4}{t} - 2 + \dfrac{3}{t} - 2 + \dfrac{2}{t} = 14$

　　$\dfrac{9}{t} \Rightarrow 18 \quad c.a.: t \neq 0 \Rightarrow t = \dfrac{1}{2} \Rightarrow \log_9 x = \dfrac{1}{2} \Rightarrow \boxed{x = 3}$

49.　$\dfrac{\log_a x + 3}{\log_a x - 3} - \dfrac{\log_a x - 3}{\log_a x + 3} = \dfrac{5 \log_a x}{\log_a^2 x - 9}$

　　$c.a.: x > 0:$　pongo: $\log_a x = t \Rightarrow \dfrac{t + 3}{t - 3} - \dfrac{t - 3}{t + 3} = \dfrac{5t}{t^2 - 9}$

　　$t^2 + 6t + 9 - t^2 + 6t - 9 = 5t \quad c.a.: t \neq \pm 3 \Rightarrow 7t = 0 \Rightarrow \log_a x = 0 \Rightarrow \boxed{x = 1}$

50.　$\left|\log_2 x + 3\right| = 5$

　　$c.a.: x > 0:$　$\log_2 x + 3 = \pm 5$

　　$\log_2 x = 2 \Rightarrow \boxed{x = 4}$　　　　　　　　　　$\log_2 x = -8 \Rightarrow \boxed{x = \dfrac{1}{2^8}}$